

Orbits, escape speed and gravitational energy

AP Physics C (Mechanics and E&M, calculus-based) · Test answers (mark scheme)

The model answers show where each mark is earned. Method marks are awarded for correct equations even when the arithmetic is wrong. Derivations must show the integral and its limits, and explanations should state cause and effect.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 7

Question 1 Field strength (2 marks)

$g \propto 1/r^2$, so $g = 9.81 / 3^2 = \mathbf{1.09 \text{ m/s}^2}$.

Marking: inverse-square idea (1); answer with unit (1).

Question 2 Low Earth orbit (3 marks)

a. $GMm/r^2 = mv^2/r$.

b. $r = 6.97 \times 10^6 \text{ m}$, $v = \sqrt{GM/r} = \mathbf{7.56 \text{ km/s}}$.

c. $T = 2\pi r/v = \mathbf{5.79 \times 10^3 \text{ s}}$ (96.6 min).

Marking: one mark per part.

Question 3 Effect of drag (2 marks)

Drag removes mechanical energy, so $E = -GMm/2r$ becomes more negative and r decreases. Since $v = \sqrt{GM/r}$, a smaller r gives a larger v . **Kinetic energy $K = -E$ rises.**

Marking: E decreases so r decreases (1); $v = \sqrt{GM/r}$ conclusion (1).

Question 4 Escape from the Moon (3 marks)

Escape needs $\frac{1}{2}mv^2 - GMm/R = 0$, so $v = \sqrt{2GM/R} = \sqrt{2 \times 6.67 \times 10^{-11} \times 7.35 \times 10^{22} / 1.74 \times 10^6} = \mathbf{2.37 \text{ km/s}}$.

Marking: energy condition (1); formula (1); answer (1).

Question 5 Energies of an orbit (3 marks)

a. $K = GMm/2r = \mathbf{1.33 \times 10^{10} \text{ J}}$.

b. $U = -GMm/r = \mathbf{-2.65 \times 10^{10} \text{ J}}$.

c. $E = K + U = \mathbf{-1.33 \times 10^{10} \text{ J}}$.

Marking: one mark per part, with the correct sign.

Question 6 Comet speeds (3 marks)

Gravity is a central force, so angular momentum about the Sun is conserved: $m v_p r_p = m v_a (4r_p)$. Hence $v_a = v_p / 4 = \mathbf{5.0 \text{ km/s}}$. The speed is smaller far away.

Marking: central force, L conserved (1); equation (1); answer (1).

Question 7 Deriving U (2 marks)

$U(r) = -\int \text{from } \infty \text{ to } r \text{ of } F \, dr = \int \text{from } \infty \text{ to } r \text{ of } GMm/r^2 \, dr = GMm[-1/r] \text{ from } \infty \text{ to } r = \mathbf{-GMm/r}$.

Marking: correct integral and limits (1); evaluation (1).

Question 8 Kepler's third law (2 marks)

$T^2 \propto r^3$, so $T \propto r^{(3/2)}$ and the ratio is $4.0^{(3/2)} = \mathbf{8.0}$.

Marking: use of $r^{(3/2)}$ (1); answer (1).