

Orbits, escape speed and gravitational energy

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet

Name: _____ Date: _____ Class: _____

These problems train the calculus-based treatment of gravitation: integrating $F = GMm/r^2$ to get $U = -GMm/r$, then applying it to orbits, escape and elliptical motion. Contexts include Earth satellites, the Moon, comets and planets of other stars.

Level 1 problems use a single formula, level 2 problems combine energy and force ideas, and level 3 problems need integration, several steps and clear reasoning, as in AP Physics C: Mechanics unit 7.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 7

Problem 1 Force on a satellite (easy)

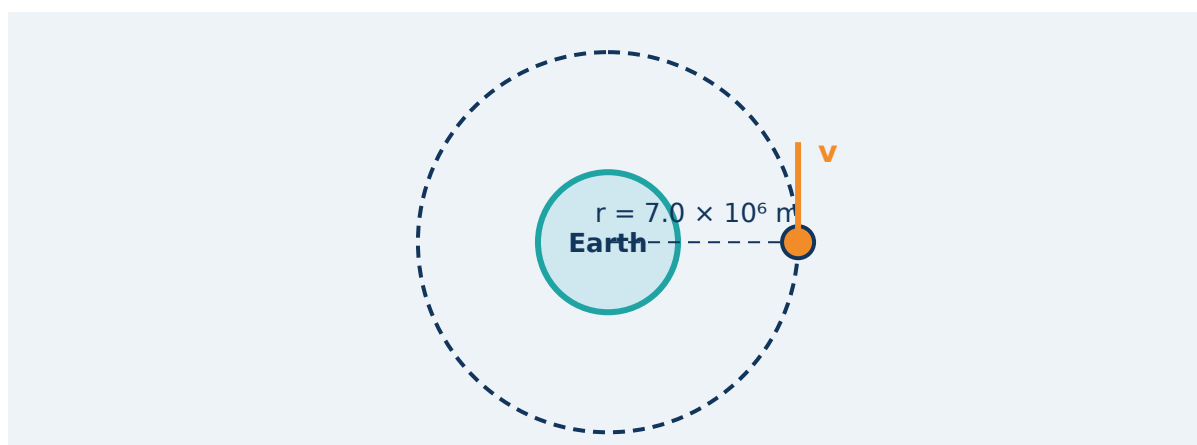
A satellite of mass 500 kg is at $r = 7.0 \times 10^6$ m from Earth's centre. Use $GM = 3.98 \times 10^{14}$ N m²/kg².

1. Calculate the gravitational force on it.
2. Calculate the gravitational field strength at that distance.

Problem 2 Potential energy (easy)

For the same 500 kg satellite at $r = 7.0 \times 10^6$ m, calculate its gravitational potential energy. State the reference point used.

Problem 3 Orbital speed and period (easy)



Problem 3: a satellite in a circular orbit.

Calculate the speed and period of the satellite in the figure.

Problem 4 Scaling Kepler's third law (easy)

A satellite orbits at radius r with period T . A second satellite orbits the same planet at radius $4r$.

1. Calculate the period of the second satellite.
2. State the ratio of their speeds.

Problem 5 Weightlessness (easy)

The International Space Station orbits at 400 km above the surface.

1. Calculate g at that height.
2. Explain why the astronauts still float.

Problem 6 Escape speed (easy)

Calculate the escape speed from Earth's surface ($R = 6.37 \times 10^6$ m) and explain why it does not depend on the mass of the rocket.

Problem 7 Energies of a circular orbit (medium)

A 1200 kg satellite is in a circular orbit of radius 8.0×10^6 m. Calculate its kinetic energy, its potential energy and its total energy.

Problem 8 Raising the orbit (medium)

A 600 kg satellite is moved from a circular orbit of radius 7.0×10^6 m to one of radius 1.0×10^7 m.

1. Calculate the change in its total energy.
2. Calculate its speed in each orbit and comment on the result.

Problem 9 Geostationary orbit (medium)

A geostationary satellite has a period of 8.62×10^4 s (one sidereal day).

1. Calculate its orbital radius.
2. Calculate its altitude above Earth's surface.
3. Calculate its speed.

Problem 10 Period of the Moon (medium)

The Moon orbits Earth at a mean radius of 3.84×10^8 m. Use Kepler's third law with $GM = 3.98 \times 10^{14}$ to calculate its period in days.

Problem 11 Escape from orbit (medium)

A spacecraft is in a circular orbit 400 km above Earth.

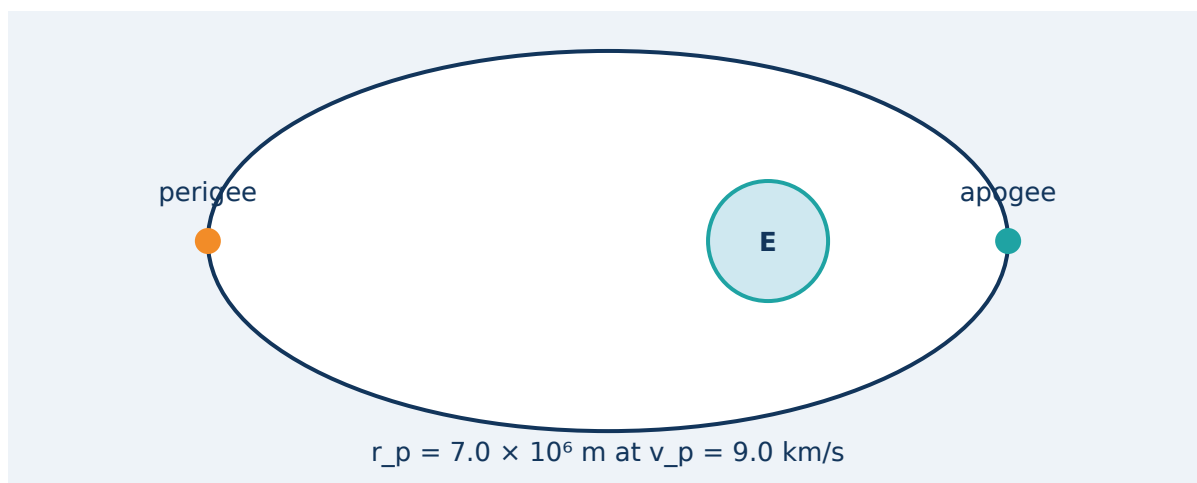
1. Calculate the escape speed at that radius.

- Calculate the extra speed it needs, along its direction of motion, to escape.

Problem 12 Energy to escape (medium)

Calculate the minimum kinetic energy needed to launch a 2.0×10^3 kg probe from Earth's surface to infinity, ignoring air resistance and Earth's rotation.

Problem 13 Perigee and apogee (medium)



Problem 13: perigee and apogee of an elliptical orbit.

A satellite has speed 9.0 km/s at perigee $r_p = 7.0 \times 10^6$ m and its apogee is at $r_a = 1.73 \times 10^7$ m.

- Calculate its speed at apogee.
- Calculate the semi-major axis and the total energy per kilogram.

Problem 14 Reading a potential energy graph (medium)



Problem 14: U/m against r/R for Earth.

The graph shows the potential energy per kilogram near Earth.

- Read the energy per kilogram needed to escape from the surface.
- Calculate the total energy per kilogram of a circular orbit at $r = 2R$.

3. Calculate the energy per kilogram needed to put a mass from rest at the surface into that orbit.

Problem 15 Spot the error (medium)

A student writes: 'Air drag removes energy from a satellite, so the satellite slows down and falls.'

1. Identify the error.
2. Give the correct description.

Problem 16 Integrating to find the work (hard)

A 1500 kg spacecraft is lifted from Earth's surface ($r = R$) to $r = 2R$.

1. Show by integration that the work done against gravity is $GMm/2R$.
2. Calculate its value.
3. Compare it with mgh using $h = R$ and explain the difference.

Problem 17 Comet orbit (hard)

A comet orbits a star with $GM = 1.33 \times 10^{20} \text{ m}^3/\text{s}^2$. Its perihelion is $1.00 \times 10^{11} \text{ m}$ and its aphelion is $2.50 \times 10^{11} \text{ m}$.

1. Calculate the semi-major axis.
2. Calculate the orbital period in years ($1 \text{ year} = 3.16 \times 10^7 \text{ s}$).
3. Calculate the speeds at perihelion and aphelion.

Problem 18 Mass of a planet from a moon (hard)

Astronomers track a small moon circling a distant giant planet. Its path is almost a circle of radius $4.22 \times 10^8 \text{ m}$ and one lap lasts 1.77 days.

1. Express the lap time in seconds.
2. Deduce the mass of the giant planet.

Problem 19 Speed at infinity (hard)

A probe is launched from Earth's surface at 15.0 km/s , well above escape speed. Ignore air resistance.

1. Write the energy equation between the surface and a very distant point.
2. Calculate the probe's speed far from Earth.

Problem 20 Falling from rest (hard)

A 250 kg object is released from rest at $r = 1.0 \times 10^7 \text{ m}$ and falls radially towards Earth's centre. Ignore air resistance.

1. Calculate the work done by gravity as it falls to $r = 7.0 \times 10^6 \text{ m}$.
2. Calculate its speed there.