

Orbits, escape speed and gravitational energy

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet answers

Each answer shows the formula, the substitution and the result with its unit in bold. Use $GM = 3.98 \times 10^{14} \text{ N m}^2/\text{kg}^2$ for Earth and $R = 6.37 \times 10^6 \text{ m}$. Always measure r from the centre of the planet, and keep signs consistent when working with negative energies.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 7

Problem 1 Force on a satellite

- $F = GMm/r^2 = (3.98 \times 10^{14} \times 500) / (7.0 \times 10^6)^2 = 1.99 \times 10^{17} / 4.9 \times 10^{13} = \mathbf{4.1 \times 10^3 \text{ N}}$.
- $g = F/m = GM/r^2 = \mathbf{8.1 \text{ m/s}^2}$.

Problem 2 Potential energy

$U = -GMm/r = -(3.98 \times 10^{14} \times 500) / (7.0 \times 10^6) = \mathbf{-2.8 \times 10^{10} \text{ J}}$. The reference point is $U = 0$ at infinite separation.

Problem 3 Orbital speed and period

$v = \sqrt{GM/r} = \sqrt{(3.98 \times 10^{14} / 7.0 \times 10^6)} = \mathbf{7.5 \text{ km/s}}$. $T = 2\pi r/v = 2\pi(7.0 \times 10^6) / 7542 = 5.83 \times 10^3 \text{ s} = \mathbf{97 \text{ min}}$.

Problem 4 Scaling Kepler's third law

- $T \propto r^{3/2}$, so $T_2 = 4^{3/2} T = \mathbf{8T}$.
- $v \propto r^{-1/2}$, so $v_2/v_1 = 1/\sqrt{4} = \mathbf{1/2}$.

Problem 5 Weightlessness

- $g = GM/r^2 = 3.98 \times 10^{14} / (6.77 \times 10^6)^2 = \mathbf{8.7 \text{ m/s}^2}$.
- The astronauts and station are in free fall together, so there is no contact force between them: **apparent weightlessness, not zero gravity**.

Problem 6 Escape speed

$v_{\text{esc}} = \sqrt{2GM/R} = \sqrt{(2 \times 3.98 \times 10^{14} / 6.37 \times 10^6)} = \mathbf{11.2 \text{ km/s}}$. In $\frac{1}{2}mv^2 = GMm/R$ the mass m cancels.

Problem 7 Energies of a circular orbit

$K = GMm/2r = (3.98 \times 10^{14} \times 1200) / (1.6 \times 10^7) = \mathbf{3.0 \times 10^{10} \text{ J}}$. $U = -2K = \mathbf{-6.0 \times 10^{10} \text{ J}}$.

$$E = K + U = -3.0 \times 10^{10} \text{ J.}$$

Problem 8 Raising the orbit

- $\Delta E = (GMm/2)(1/r_1 - 1/r_2) = (3.98 \times 10^{14} \times 600 / 2)(1.4286 \times 10^{-7} - 1.0 \times 10^{-7}) = 5.1 \times 10^9 \text{ J.}$
- $v_1 = \sqrt{GM/r_1} = 7.54 \text{ km/s}$ and $v_2 = 6.31 \text{ km/s}$. The speed **decreases** although the energy increases, because U rises by twice the amount that K falls.

Problem 9 Geostationary orbit

- $r^3 = GM T^2/4\pi^2 = 3.98 \times 10^{14} \times (8.62 \times 10^4)^2 / 39.48 = 7.49 \times 10^{22}$, so $r = 4.22 \times 10^7 \text{ m.}$
- Altitude $= 4.22 \times 10^7 - 6.37 \times 10^6 = 3.58 \times 10^7 \text{ m.}$
- $v = 2\pi r/T = 3.07 \text{ km/s.}$

Problem 10 Period of the Moon

$$T = 2\pi\sqrt{(r^3/GM)} = 2\pi\sqrt{(5.66 \times 10^{25} / 3.98 \times 10^{14})} = 2\pi \times 3.77 \times 10^5 = 2.37 \times 10^6 \text{ s} = 27.4 \text{ days.}$$

Problem 11 Escape from orbit

- $v_{\text{esc}} = \sqrt{2GM/r} = \sqrt{(2 \times 3.98 \times 10^{14} / 6.77 \times 10^6)} = 10.8 \text{ km/s.}$
- The orbital speed is 7.67 km/s , so $\Delta v = 10.85 - 7.67 = 3.2 \text{ km/s.}$

Problem 12 Energy to escape

$$E_{\text{min}} = GMm/R = 3.98 \times 10^{14} \times 2.0 \times 10^3 / 6.37 \times 10^6 = 1.25 \times 10^{11} \text{ J.}$$

Problem 13 Perigee and apogee

- Angular momentum is conserved: $v_a = v_p r_p / r_a = 9000 \times 7.0 \times 10^6 / 1.73 \times 10^7 = 3.6 \text{ km/s.}$
- $a = (r_p + r_a)/2 = 1.2 \times 10^7 \text{ m.}$ $E/m = -GM/2a = -3.98 \times 10^{14} / 2.43 \times 10^7 = -1.6 \times 10^7 \text{ J/kg.}$

Problem 14 Reading a potential energy graph

- 62.5 MJ/kg** (the surface value is $U/m = -GM/R = -62.5 \text{ MJ/kg}$).
- $E/m = -GM/(2 \times 2R) = -62.5 / 4 = -15.6 \text{ MJ/kg.}$
- $\Delta E/m = -15.6 - (-62.5) = 46.9 \text{ MJ/kg.}$

Problem 15 Spot the error

- A circular orbit has $K = -E$, so losing energy increases K .

2. **Drag lowers the orbit and the satellite speeds up:** $v = \sqrt{GM/r}$ grows as r falls; the extra kinetic energy comes from gravitational potential energy.

Problem 16 Integrating to find the work

1. $W = \int \text{from } R \text{ to } 2R \text{ of } GMm/r^2 \, dr = GMm(1/R - 1/2R) = \mathbf{GMm/2R}.$
2. $W = 3.98 \times 10^{14} \times 1500 / (2 \times 6.37 \times 10^6) = \mathbf{4.7 \times 10^{10} \, J}.$
3. $mgh = 1500 \times 9.81 \times 6.37 \times 10^6 = 9.4 \times 10^{10} \, J$, twice as large, because g falls with height and is not constant.

Problem 17 Comet orbit

1. $a = (1.00 + 2.50) \times 10^{11} / 2 = \mathbf{1.75 \times 10^{11} \, m}.$
2. $T = 2\pi\sqrt{a^3/GM} = 2\pi\sqrt{(5.36 \times 10^{33} / 1.33 \times 10^{20})} = 3.99 \times 10^7 \, s = \mathbf{1.26 \, years}.$
3. $v^2 = GM(2/r - 1/a)$: at perihelion $v = \mathbf{4.36 \times 10^4 \, m/s}$; by angular momentum $v_a = 4.36 \times 10^4 \times 1.00 / 2.50 = \mathbf{1.74 \times 10^4 \, m/s}.$

Problem 18 Mass of a planet from a moon

1. $T = 1.77 \times 86\,400 = \mathbf{1.53 \times 10^5 \, s}.$
2. $M = 4\pi^2 r^3 / (G T^2) = 39.48 \times 7.515 \times 10^{25} / (6.67 \times 10^{-11} \times 2.339 \times 10^{10}) = \mathbf{1.9 \times 10^{27} \, kg}.$

Problem 19 Speed at infinity

1. $\frac{1}{2}mv_0^2 - GMm/R = \frac{1}{2}m v_\infty^2.$
2. $v_\infty^2 = v_0^2 - 2GM/R = 2.25 \times 10^8 - 1.25 \times 10^8 = 1.00 \times 10^8$, so $v_\infty = \mathbf{10 \, km/s}.$

Problem 20 Falling from rest

1. $W = \int F \, dr = GMm(1/r_2 - 1/r_1) = 3.98 \times 10^{14} \times 250 \times (1.4286 \times 10^{-7} - 1.0 \times 10^{-7}) = \mathbf{4.3 \times 10^9 \, J}.$
2. $\frac{1}{2}mv^2 = W$, so $v = \sqrt{(2 \times 4.27 \times 10^9 / 250)} = \mathbf{5.8 \, km/s}.$