

Projectiles and relative motion in a plane

AP Physics 1 (algebra-based) · Lesson

Quick answer: Projectile motion is two-dimensional motion under gravity alone. The horizontal velocity stays constant because $a_x = 0$, while the vertical velocity changes at 9.81 m/s^2 downward. Splitting a velocity into components lets you solve each direction separately and link them with the shared time.

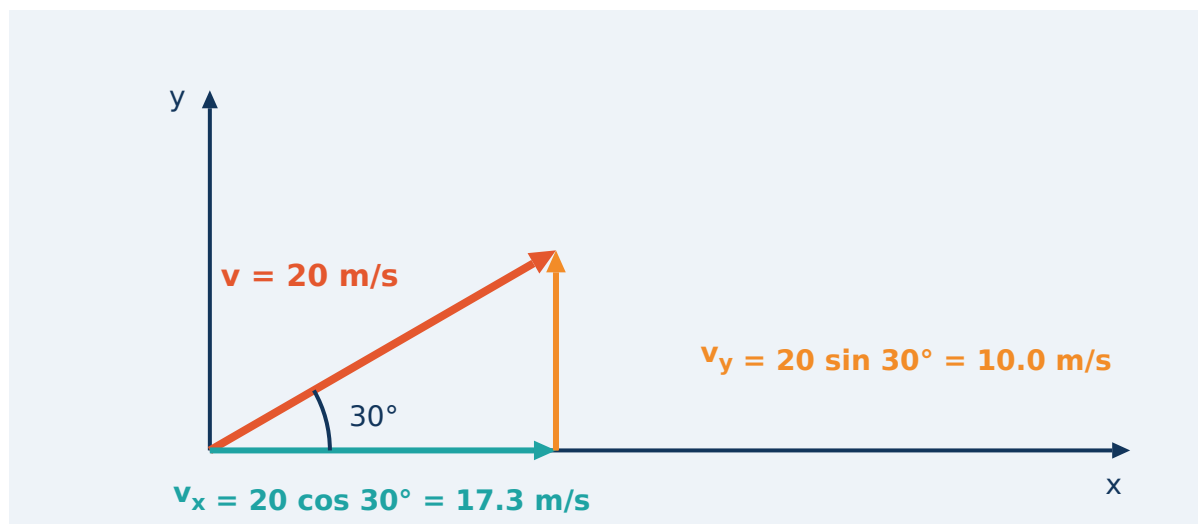
Level: AP Physics 1 (algebra-based) · **Curriculum:** AP P1 Unit 1

How do you describe motion in two dimensions?

Motion in a plane needs two numbers at every instant. The trick used in AP Physics 1 unit 1 is to split every vector into a horizontal part and a vertical part, solve each direction as a separate one-dimensional problem, and recombine the answers at the end.

Definition

A **vector component** is the projection of a vector on one axis. For a speed v at angle θ above the horizontal, $v_x = v \cos \theta$ and $v_y = v \sin \theta$. A **projectile** is an object moving only under gravity, with air resistance ignored.



A velocity vector is replaced by two perpendicular components.

How does a projectile move?

Gravity pulls only downward, so the horizontal motion has no acceleration and the vertical motion has a constant downward acceleration. The two motions share only the clock: the time t is the same in both.

Key law

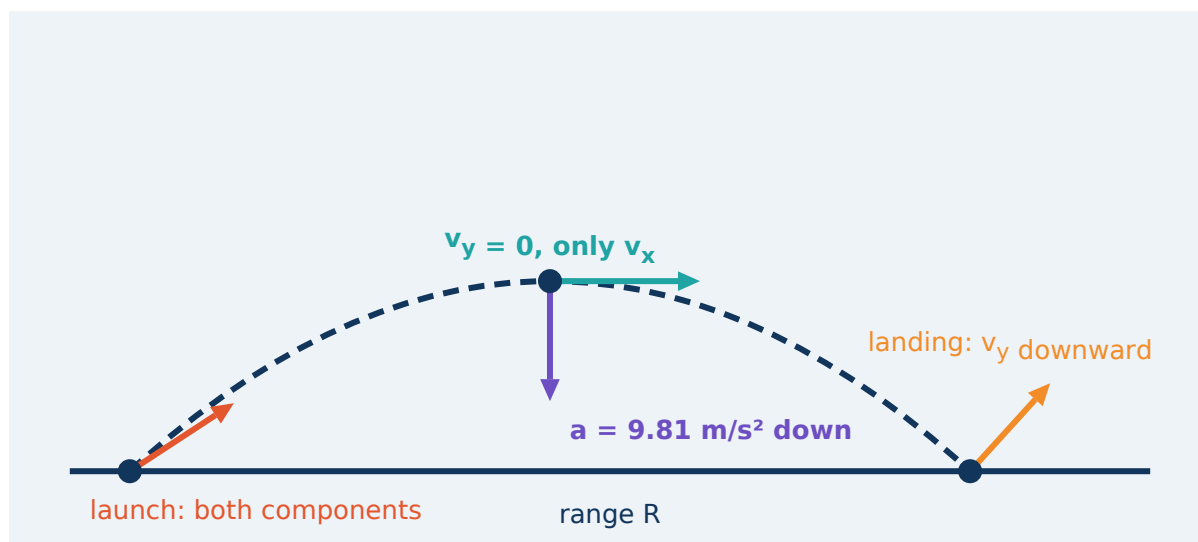
Horizontal: $a_x = 0$, so v_x is constant. Vertical: $a_y = -g = -9.81 \text{ m/s}^2$ at every point of the

flight, including the top.

Formula

$$\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_x \mathbf{t} \quad \mathbf{v}_y = \mathbf{v}_{y0} - \mathbf{g} \mathbf{t} \quad \mathbf{y} = \mathbf{y}_0 + \mathbf{v}_{y0} \mathbf{t} - \frac{1}{2} \mathbf{g} \mathbf{t}^2$$

x, y = position (m); v = velocity (m/s); $g = 9.81 \text{ m/s}^2$; t = time (s). Up is positive.



Horizontal velocity stays constant; the acceleration is downward everywhere on the path (not to scale).

Quantity	Horizontal (x)	Vertical (y)
Acceleration	0	-9.81 m/s^2
Velocity	constant	changes by -9.81 m/s every second
At the top of the path	v_x unchanged	$v_y = 0$

Worked example

Aarav in Pune throws a stone horizontally at 12 m/s from a bridge 19.6 m above a canal.

Vertical: $19.6 = \frac{1}{2} \times 9.81 \times t^2$, so $t = \sqrt{(2 \times 19.6 \div 9.81)} = \mathbf{2.0 \text{ s}}$.

Horizontal: $x = 12 \times 2.0 = \mathbf{24 \text{ m}}$ from the base of the bridge.

On impact $v_y = 9.81 \times 2.0 = 19.6 \text{ m/s}$ downward, so the speed is $\sqrt{(12^2 + 19.6^2)} = \mathbf{23 \text{ m/s}}$.

Worked example

Mei in Singapore kicks a football from level ground at 20 m/s , 30° above the horizontal.

$v_x = 17.3 \text{ m/s}$ and $v_{y0} = 10.0 \text{ m/s}$.

Time up: $10.0 \div 9.81 = 1.02 \text{ s}$, so the time of flight is $\mathbf{2.04 \text{ s}}$.

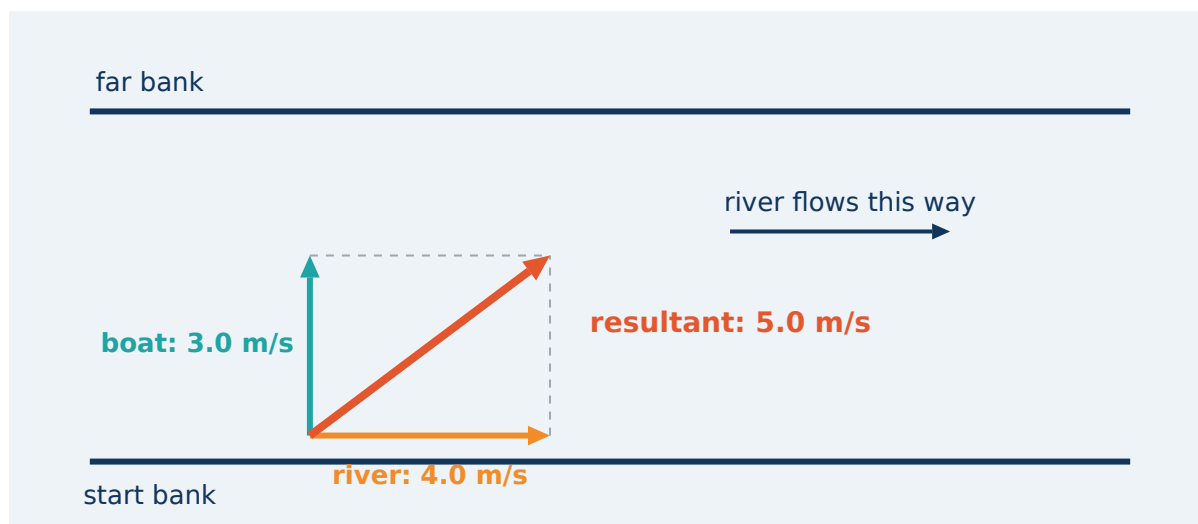
Range = $17.3 \times 2.04 = \mathbf{35.3 \text{ m}}$. Maximum height = $10.0^2 \div (2 \times 9.81) = \mathbf{5.10 \text{ m}}$.

Common mistake

Many students say the ball has zero acceleration at the top because its vertical velocity is zero. The velocity is passing through zero, but the acceleration is still 9.81 m/s^2 downward, which is why the ball turns around.

How do you add velocities in relative motion?

If a boat moves relative to the water and the water moves relative to the ground, the velocity of the boat relative to the ground is the vector sum: $v_{\text{boat/ground}} = v_{\text{boat/water}} + v_{\text{water/ground}}$. Components along perpendicular directions do not affect each other.



Velocities add as vectors: the boat is carried downstream while it crosses.

Worked example

Wanjiru steers a boat straight across a 60 m wide river at 3.0 m/s relative to the water. The river flows at 4.0 m/s.

Crossing time = $60 \div 3.0 = 20 \text{ s}$. Drift downstream = $4.0 \times 20 = 80 \text{ m}$.

Speed relative to the ground = $\sqrt{(3.0^2 + 4.0^2)} = 5.0 \text{ m/s}$, at 53° from the straight-across line.

Lina's tip

Draw two columns, x and y, and list what you know in each. Only the time t appears in both columns, so find t in the easier column first, then carry it across.

Key points to remember

- Components: $v_x = v \cos \theta$, $v_y = v \sin \theta$.
- Horizontal: constant velocity; vertical: acceleration of 9.81 m/s^2 downward at every point.
- Fall time from rest vertically: $t = \sqrt{(2h \div g)}$.
- Range on level ground: $R = v^2 \sin 2\theta \div g$, greatest at 45° .
- Relative velocities add as vectors.

Projectiles and relative motion in a plane: frequently asked questions

Why is the horizontal velocity of a projectile constant?

No horizontal force acts once the object is released if air resistance is ignored. With zero net horizontal force the acceleration is zero, so the horizontal velocity cannot change.

Is the acceleration zero at the top of the flight?

No. At the top the vertical velocity is zero for an instant, but the acceleration is still 9.81 m/s^2 downward. That acceleration is what makes the ball turn around and fall.

Which launch angle gives the longest range?

On level ground with a fixed launch speed the range is greatest at 45° . Angles that add up to 90° , such as 30° and 60° , give the same range.

How do you solve a projectile problem step by step?

Resolve the initial velocity into components, write the x and y equations separately, find the time from the vertical equation, then use that time in the horizontal equation. Check signs by choosing up as positive.

How is relative motion handled in AP Physics 1?

Velocities are added as vectors: the velocity of an object relative to the ground equals its velocity relative to the medium plus the velocity of the medium relative to the ground. Perpendicular parts are treated separately.