

# Projectiles and relative motion in a plane

## AP Physics 1 (algebra-based) · Worksheet answers

Each answer shows the formula, the substitution and the final result in bold with its unit.  
Compare your method as well as your numbers, because AP scoring rewards the reasoning.

**Level:** AP Physics 1 (algebra-based) · **Curriculum:** AP P1 Unit 1

### Problem 1 Components of a bat strike

1.  $v_x = 30 \cos 20^\circ = \mathbf{28.2 \text{ m/s}}$ .
2.  $v_y = 30 \sin 20^\circ = \mathbf{10.3 \text{ m/s}}$ .
3. The horizontal component stays constant because  $a_x = 0$ .

### Problem 2 True or false at the top

1. True:  $v_y = 0$  at the top.
2. False:  $v_x$  is unchanged.
3. False.
4. True:  $\mathbf{a = 9.81 \text{ m/s}^2 \text{ downward}}$  throughout the flight.

### Problem 3 Walk across a field

1. Distance =  $40 + 30 = \mathbf{70 \text{ m}}$ .
2. Displacement =  $\sqrt{40^2 + 30^2} = \sqrt{2500} = \mathbf{50 \text{ m}}$ .
3.  $\tan \theta = 30 \div 40 = 0.75$ , so  $\theta = \mathbf{36.9^\circ \text{ north of east}}$ .

### Problem 4 Marble leaving a table

1.  $0.80 = \frac{1}{2} \times 9.81 \times t^2$ , so  $t = \sqrt{(1.6 \div 9.81)} = \mathbf{0.40 \text{ s}}$ .
2.  $x = 1.5 \times 0.404 = \mathbf{0.61 \text{ m}}$ .

### Problem 5 Dropped or launched

1. **They land together.**
2. Both start with zero vertical velocity and have the same vertical acceleration and the same height, so  $y = \frac{1}{2}gt^2$  gives the same time. The horizontal motion does not affect the vertical motion.

### Problem 6 Football over level ground

1.  $v_x = 25 \cos 40^\circ = 19.2 \text{ m/s}$ ;  $v_{y0} = 25 \sin 40^\circ = 16.1 \text{ m/s}$ .
2.  $T = 2 \times 16.07 \div 9.81 = \mathbf{3.28 \text{ s}}$ .

3.  $R = 19.15 \times 3.276 = \mathbf{62.7 \text{ m}}$ .
4.  $H = 16.07^2 \div (2 \times 9.81) = \mathbf{13.2 \text{ m}}$ .

### Problem 7 Reading a velocity graph

1. The graph crosses zero at **1.5 s**.
2. Gradient =  $(-14.7 - 14.7) \div 3.0 = \mathbf{-9.8 \text{ m/s}^2}$ ; it is the acceleration, equal to  $-g$  (within rounding).
3. Area of the triangle =  $\frac{1}{2} \times 1.5 \times 14.7 = \mathbf{11 \text{ m}}$ .
4. The ball lands at **3.0 s**, when  $v_y = -14.7 \text{ m/s}$ .

### Problem 8 Stone thrown from a cliff

1.  $45 = \frac{1}{2} \times 9.81 \times t^2$ ,  $t = \sqrt{(90 \div 9.81)} = \mathbf{3.03 \text{ s}}$ .
2.  $x = 8.0 \times 3.03 = \mathbf{24 \text{ m}}$ .
3.  $v_y = 9.81 \times 3.03 = 29.7 \text{ m/s}$ ; speed =  $\sqrt{(8.0^2 + 29.7^2)} = \mathbf{30.8 \text{ m/s}}$ .
4.  $\tan \theta = 29.7 \div 8.0$ , so  $\theta = \mathbf{75^\circ \text{ below the horizontal}}$ .

### Problem 9 Moving walkway

1.  $1.4 + 0.80 = \mathbf{2.2 \text{ m/s}}$ .
2.  $t = 66 \div 2.2 = \mathbf{30 \text{ s}}$ .
3.  $1.4 - 0.80 = \mathbf{0.60 \text{ m/s}}$  in his walking direction.

### Problem 10 Spot the error

1.  $v_x = 20 \cos 60^\circ = 10 \text{ m/s}$ .
2. At the top  $v_y = 0$ , so the speed equals  $v_x = \mathbf{10 \text{ m/s}}$ .
3. Only the vertical component is zero at the top. The horizontal component never changes, so the speed is not zero.

### Problem 11 Free kick and crossbar

1.  $v_x = 18 \cos 35^\circ = 14.75 \text{ m/s}$ ;  $t = 20 \div 14.75 = \mathbf{1.36 \text{ s}}$ .
2.  $v_{y0} = 18 \sin 35^\circ = 10.32 \text{ m/s}$ ;  $y = 10.32 \times 1.356 - \frac{1}{2} \times 9.81 \times 1.356^2 = 14.0 - 9.0 = \mathbf{5.0 \text{ m}}$ .
3.  $5.0 \text{ m}$  is above  $2.44 \text{ m}$ , so the ball **passes over the bar**.

### Problem 12 Same speed, different angles

1.  $R = 15^2 \times \sin 90^\circ \div 9.81 = \mathbf{22.9 \text{ m}}$ .
2.  $R = 225 \times \sin 60^\circ \div 9.81 = \mathbf{19.9 \text{ m}}$ .
3.  $R = 225 \times \sin 120^\circ \div 9.81 = \mathbf{19.9 \text{ m}}$ . Angles that add up to  $90^\circ$  give the same range, because  $\sin 2\theta$  is equal for both.

### Problem 13 River crossing

1.  $t = 80 \div 2.0 = \mathbf{40\ s}$ .
2. Drift =  $1.5 \times 40 = \mathbf{60\ m}$ .
3.  $v = \sqrt{(2.0^2 + 1.5^2)} = \mathbf{2.5\ m/s}$ .

### Problem 14 Reading a strobe diagram

1. Gravity acts only vertically, so  $a_x = 0$  and the ball covers equal distances in equal times:  $10 \times 0.5 = \mathbf{5\ m}$  per interval.
2.  $y = \frac{1}{2} \times 9.81 \times 1.0^2 = \mathbf{4.9\ m}$ , which matches the diagram (4.91 m).
3.  $y = \frac{1}{2} \times 9.81 \times 2.5^2 = \mathbf{31\ m}$ .

### Problem 15 Ball hitting a wall

1.  $v_x = 20 \cos 50^\circ = 12.86\ \text{m/s}$ ;  $t = 18 \div 12.86 = \mathbf{1.40\ s}$ .
2.  $v_{y0} = 20 \sin 50^\circ = 15.32\ \text{m/s}$ ;  $y = 15.32 \times 1.40 - 4.905 \times 1.40^2 = 21.4 - 9.6 = \mathbf{11.8\ m}$ .
3.  $v_y = 15.32 - 9.81 \times 1.40 = +1.6\ \text{m/s}$ , positive, so the ball is **still rising**.

### Problem 16 Stone thrown from a bridge

1. Take up as positive and the water as  $y = -12\ \text{m}$ :  $-12 = 7.5t - 4.905t^2$  ( $v_{y0} = 15 \sin 30^\circ = 7.5\ \text{m/s}$ ). So  $4.905t^2 - 7.5t - 12 = 0$ , giving  $t = (7.5 + 17.08) \div 9.81 = \mathbf{2.51\ s}$ .
2.  $v_x = 15 \cos 30^\circ = 12.99\ \text{m/s}$ ;  $x = 12.99 \times 2.506 = \mathbf{32.5\ m}$ .
3.  $v_y = 7.5 - 9.81 \times 2.506 = -17.1\ \text{m/s}$ ; speed =  $\sqrt{(12.99^2 + 17.08^2)} = \mathbf{21.5\ m/s}$  at  $\tan^{-1}(17.08 \div 12.99) = \mathbf{53^\circ}$  below the horizontal.

### Problem 17 Heading into the wind

1. Ground velocity components:  $15\ \text{m/s}$  east and  $60\ \text{m/s}$  north, so speed =  $\sqrt{(15^2 + 60^2)} = \mathbf{62\ m/s}$ , at  $\tan^{-1}(15 \div 60) = \mathbf{14^\circ}$  east of north.
2. The plane must cancel the  $15\ \text{m/s}$  eastward drift:  $\sin \theta = 15 \div 60 = 0.25$ , so  $\theta = \mathbf{14.5^\circ}$  west of north.
3. Ground speed =  $\sqrt{(60^2 - 15^2)} = \mathbf{58\ m/s}$  due north.

### Problem 18 Basketball from its peak

1.  $v_{y0} = \sqrt{(2gH)} = \sqrt{(2 \times 9.81 \times 3.2)} = \mathbf{7.9\ m/s}$ .
2.  $v = 7.92 \div \sin 60^\circ = \mathbf{9.1\ m/s}$ .
3.  $T = 2 \times 7.92 \div 9.81 = 1.62\ \text{s}$ ;  $v_x = 9.15 \cos 60^\circ = 4.57\ \text{m/s}$ ;  $R = 4.57 \times 1.62 = \mathbf{7.4\ m}$ .

### Problem 19 Analysing tracking data

1.  $v_x = 2.4 \div 0.2 = \mathbf{12\ m/s}$  (constant for every row).
2.  $y \div t = 8.0, 7.05, 6.05, 5.08\ \text{m/s}$ . Since  $y \div t = v_{y0} - \frac{1}{2}gt$ , a line through these points

extrapolates to  $t = 0$  at about **9.0 m/s**. The gradient ( $-4.9 \text{ m/s}^2$ ) confirms  $\frac{1}{2}g$ .

3.  $v = \sqrt{12^2 + 9.0^2} = \mathbf{15 \text{ m/s}}$ ;  $\theta = \tan^{-1}(9.0 \div 12) = \mathbf{37^\circ}$ .

4.  $t = 9.0 \div 9.81 = \mathbf{0.92 \text{ s}}$ .

### Problem 20 Planning a launcher test

1. Clamp the launcher at the bench edge, fire the ball several times, and use a plumb line to mark the point on the floor below the edge. Lay white paper with carbon paper on top where the ball lands, and measure the horizontal range  $x$  and the bench height  $h$  with the rule.

2.  $t = \sqrt{2h \div g}$  and  $v = x \div t$ , so  **$v = x \sqrt{g \div 2h}$** .

3. Repeat at least ten times and average the range; tape the paper down; measure from the plumb line to the centre of the cluster of marks.

4. Fall time would increase by a factor  $\sqrt{2}$ , so the range would be  **$\sqrt{2}$  times larger** ( $1.41\times$ ), while the launch speed stays the same.