

Independent horizontal and vertical motion of projectiles

Physics grades 11-12 (A Level · IB · Class 11-12) · Lesson

Quick answer: Projectile motion combines a constant horizontal velocity with a vertical motion of constant acceleration $g = 9.81 \text{ m/s}^2$ downwards. The two directions are treated separately and linked only by the time of flight. On level ground, without air resistance, range $R = u^2 \sin 2\theta \div g$, which is greatest at 45° .

Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** A Level 3.4

What is projectile motion?

A **projectile** is an object that moves through the air under the action of gravity alone once it has been launched. A kicked football, a thrown javelin and a jet of water from a hose all follow a curved path because the horizontal and vertical parts of the motion behave in completely different ways.

Definition

Projectile motion is the combination of a horizontal motion at constant velocity and a vertical motion with a constant downward acceleration of $g = 9.81 \text{ m/s}^2$, when air resistance is ignored.

Why can you treat the two directions separately?

Gravity acts only downwards, so there is no horizontal force and no horizontal acceleration. The vertical motion is ordinary uniform acceleration. The two motions share only the same time of flight, so time is the link between them.



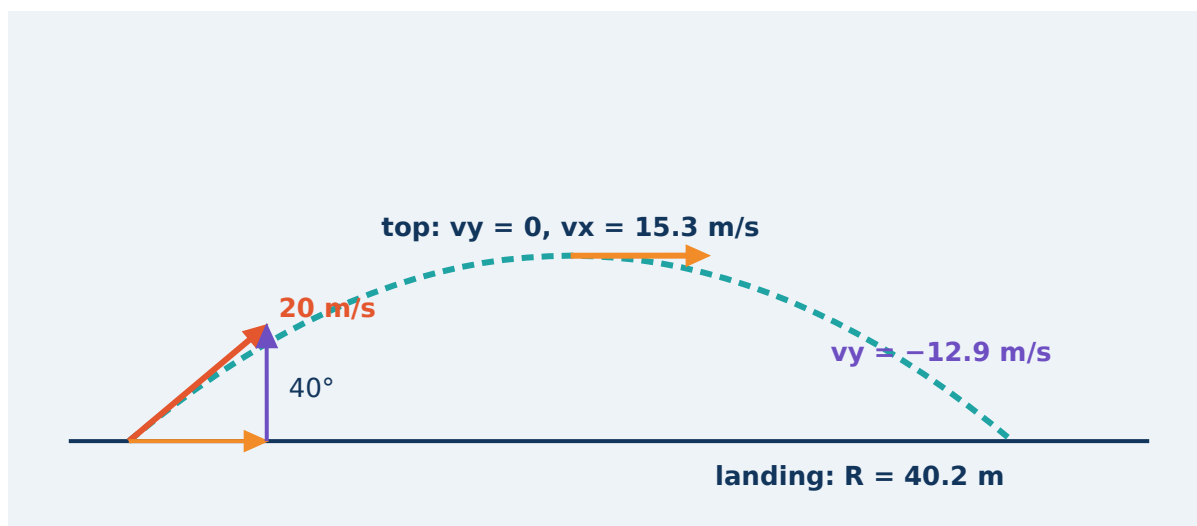
A dropped ball and a ball projected horizontally fall together.

Key law

Horizontal: $a_x = 0$, so $x = u_x t$. Vertical: $a_y = -g$, so use the suvat equations with u_y , g and t . Time t is the same for both directions.

How do you find the launch components?

For a launch speed u at an angle θ above the horizontal, the components are $u_x = u \cos \theta$ and $u_y = u \sin \theta$. The horizontal component never changes. The vertical component falls at 9.81 m/s every second, reaches zero at the top, and becomes negative on the way down.



Path of a ball launched at 20 m/s and 40°.

Worked example 1

A stunt rider leaves a platform 3.0 m high at 12 m/s horizontally. Vertical: $3.0 = \frac{1}{2} \times 9.81 \times t^2$, so $t = \sqrt{(6.0 \div 9.81)} = 0.78 \text{ s}$. Horizontal range = $12 \times 0.782 = 9.4 \text{ m}$. Vertical speed on landing = $9.81 \times 0.782 = 7.7 \text{ m/s}$. The rider lands **9.4 m** from the platform after **0.78 s**.

How do you calculate time of flight, range and height?

On level ground, the time of flight is $T = 2u_y \div g$, the maximum height is $H = u_y^2 \div 2g$ and the range is $R = u_x T = u^2 \sin 2\theta \div g$.

Worked example 2

A football is kicked at 20 m/s at 40° above the ground. $u_x = 20 \cos 40^\circ = 15.3 \text{ m/s}$ and $u_y = 20 \sin 40^\circ = 12.9 \text{ m/s}$. $T = 2 \times 12.86 \div 9.81 = 2.62 \text{ s}$. $R = 15.32 \times 2.62 = 40.2 \text{ m}$. $H = 12.86^2 \div (2 \times 9.81) = 8.42 \text{ m}$. The ball flies for **2.62 s**, lands **40.2 m** away and peaks at **8.42 m**.

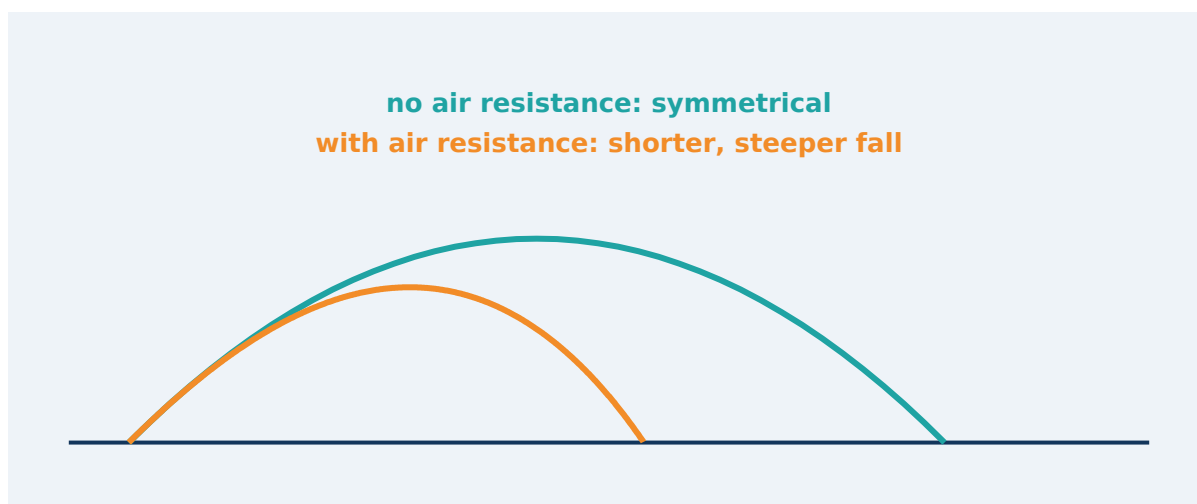
Launch angle	Range for $u = 20 \text{ m/s}$	Comment
15°	20.4 m	low and flat
30°	35.3 m	same range as 60°

Launch angle	Range for $u = 20 \text{ m/s}$	Comment
45°	40.8 m	maximum range
60°	35.3 m	same range as 30°
75°	20.4 m	high and steep

The table shows that angles which add to 90° give the same range, and that 45° gives the largest range on level ground when air resistance is ignored.

How does air resistance change the path?

Real balls experience a drag force that points opposite to the velocity. Qualitatively, drag removes horizontal velocity too, so the range is shorter and the maximum height is lower. The path is no longer symmetrical: the descent is steeper than the climb, the landing speed is less than the launch speed, and the best angle for range falls below 45° .



Drag shortens the range and makes the descent steeper.

Common mistake

Students often think that a heavier ball falls faster or that the horizontal velocity decreases because the ball slows down in the air. Without air resistance, mass does not matter and the horizontal velocity stays exactly the same until landing.

Lina's tip

Split the problem into two columns, x and y, and write time at the top. First find t from whichever column has enough data, then carry it across to the other column.

Key points to remember

- Horizontal: constant velocity, $a_x = 0$
- Vertical: $a_y = -9.81 \text{ m/s}^2$ (suvat equations)
- Components: $u_x = u \cos \theta$, $u_y = u \sin \theta$
- Level ground: $T = 2u_y \div g$, $H = u_y^2 \div 2g$, $R = u^2 \sin 2\theta \div g$
- Air resistance shortens the range and steepens the descent

Independent horizontal and vertical motion of projectiles: frequently asked questions

Why is the horizontal velocity constant in projectile motion?

Once the object has left the launcher, the only force acting is gravity, which is vertical. With no horizontal force there is no horizontal acceleration, so the horizontal velocity stays the same. Air resistance would change this in real life.

What is the velocity at the top of the path?

The vertical component is zero, but the horizontal component is still u_x . The speed at the top is therefore the smallest speed on the path. The acceleration at the top is still 9.81 m/s^2 downwards.

Which launch angle gives the maximum range?

On level ground and without air resistance, 45° gives the greatest range, because $\sin 2\theta$ is then 1. Angles that add up to 90° give equal ranges. With air resistance the best angle is a little lower.

Does the mass of a projectile affect its path?

Without air resistance it does not, because the acceleration is g for every mass. In real situations a light object such as a feather is affected more by drag than a dense object such as a metal ball.

How does air resistance affect the range?

Drag opposes the motion, so it reduces both velocity components. The ball rises to a lower height, lands closer and falls at a steeper angle than the ideal parabola. The path is no longer symmetrical.