

Independent horizontal and vertical motion of projectiles

Physics grades 11-12 (A Level · IB · Class 11-12) · Worksheet answers

Each answer separates the horizontal and the vertical motion and shows the formula, the substitution and the result in bold. Intermediate values are kept unrounded, so your last digit may differ slightly. Compare your method as well as your numbers.

Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** A Level 3.4

Problem 1 True or false

1. **True:** no horizontal force acts.
2. **True:** the vertical component changes sign there.
3. **False:** the acceleration is 9.81 m/s^2 downwards everywhere.
4. **False:** mass does not affect the motion without air resistance.

Problem 2 Launch components

1. $35 \cos 25^\circ = 35 \times 0.906 = \mathbf{31.7 \text{ m/s}}$.
2. $35 \sin 25^\circ = 35 \times 0.423 = \mathbf{14.8 \text{ m/s}}$.

Problem 3 Stone from a bridge

1. $t = \sqrt{(2 \times 20 \div 9.81)} = \sqrt{4.077} = \mathbf{2.02 \text{ s}}$.
2. **2.02 s:** the vertical motion starts from zero vertical velocity for both stones and does not depend on the horizontal velocity.

Problem 4 Garden hose

1. $t = \sqrt{(2 \times 0.80 \div 9.81)} = \sqrt{0.1631} = \mathbf{0.40 \text{ s}}$.
2. $x = 4.0 \times 0.404 = \mathbf{1.6 \text{ m}}$.

Problem 5 Strobe picture

1. At every flash the two balls are at the same height, so **their vertical motions are identical**.
2. The horizontal spacing is equal, so the horizontal velocity is **constant**.

Problem 6 Vertical component and flight time

1. $T = 2u_y \div g = 29.4 \div 9.81 = \mathbf{3.0 \text{ s}}$.

2. $H = u_y^2 \div 2g = 216.1 \div 19.62 = \mathbf{11.0 \text{ m}}$.

Problem 7 A six in cricket

1. $u_x = 28 \cos 35^\circ = \mathbf{22.9 \text{ m/s}}$; $u_y = 28 \sin 35^\circ = \mathbf{16.1 \text{ m/s}}$.

2. $T = 2 \times 16.06 \div 9.81 = \mathbf{3.27 \text{ s}}$.

3. $R = 22.94 \times 3.274 = \mathbf{75.1 \text{ m}}$, which is more than 70 m, so the ball lands beyond the rope.

Problem 8 Sprinkler jet

1. $t = \sqrt{(3.6 \div 9.81)} = \mathbf{0.61 \text{ s}}$.

2. $x = 6.0 \times 0.606 = \mathbf{3.6 \text{ m}}$.

3. $v_y = 9.81 \times 0.606 = 5.94 \text{ m/s}$; $v = \sqrt{(6.0^2 + 5.94^2)} = \mathbf{8.4 \text{ m/s}}$; angle = $\tan^{-1}(5.94 \div 6.0) = \mathbf{44.7^\circ}$.

Problem 9 Kick from a wall

1. $t = \sqrt{(4.0 \div 9.81)} = \mathbf{0.64 \text{ s}}$.

2. $u = 6.0 \div 0.6386 = \mathbf{9.4 \text{ m/s}}$.

Problem 10 Complementary angles

1. $R = u^2 \sin 2\theta \div g = 625 \times \sin 60^\circ \div 9.81 = \mathbf{55.2 \text{ m}}$.

2. $\mathbf{60^\circ}$, because $30^\circ + 60^\circ = 90^\circ$.

3. The 60° throw goes higher and stays in the air **longer**.

Problem 11 Maximum range

1. At 45° , $R = 38^2 \div 9.81 = 1444 \div 9.81 = \mathbf{147 \text{ m}}$.

2. $R = 147.2 \times \sin 60^\circ = \mathbf{127 \text{ m}}$.

3. $(147.2 - 127.5) \div 147.2 = \mathbf{13 \%}$.

Problem 12 Effect of drag

1. The range is **shorter** and the maximum height is **lower** (the path is not symmetrical).

2. Drag removes horizontal velocity all the way, while gravity keeps adding downward velocity, so **the horizontal part is smaller on the way down**.

Problem 13 Archery

1. $t = 30 \div 60 = \mathbf{0.50 \text{ s}}$.

2. Drop = $\frac{1}{2} \times 9.81 \times 0.25 = \mathbf{1.2 \text{ m}}$.

3. Aim about **1.2 m above** the target centre.

Problem 14 Velocity after one second

- $u_x = 22 \cos 50^\circ = 14.1 \text{ m/s}$. $v_y = 22 \sin 50^\circ - 9.81 \times 1.0 = 16.85 - 9.81 = \mathbf{7.04 \text{ m/s}}$.
- $v = \sqrt{(14.14^2 + 7.04^2)} = \mathbf{15.8 \text{ m/s}}$.
- $\tan^{-1}(7.04 \div 14.14) = \mathbf{26.5^\circ}$.

Problem 15 Over the crossbar

- $u_x = 18 \cos 40^\circ = 13.79 \text{ m/s}$, so $t = 20 \div 13.79 = \mathbf{1.45 \text{ s}}$.
- $y = 11.57 \times 1.4505 - 4.905 \times 1.4505^2 = 16.78 - 10.32 = \mathbf{6.5 \text{ m}}$.
- 6.5 m is greater than 2.44 m, so the ball **passes over the bar**.

Problem 16 Ball against a wall

- $u_x = 7.5 \text{ m/s}$, so $t = 8.0 \div 7.5 = \mathbf{1.07 \text{ s}}$.
- $y = 12.99 \times 1.0667 - 4.905 \times 1.0667^2 = 13.86 - 5.58 = \mathbf{8.3 \text{ m}}$.
- $v_y = 12.99 - 9.81 \times 1.0667 = +2.5 \text{ m/s}$, so the ball is still **rising**.

Problem 17 Ball from a cliff

- $u_y = 10.0 \text{ m/s}$, $u_x = 17.32 \text{ m/s}$. $-30 = 10t - 4.905t^2$: $t = (10 + \sqrt{688.6}) \div 9.81 = \mathbf{3.69 \text{ s}}$.
- $x = 17.32 \times 3.694 = \mathbf{64.0 \text{ m}}$.
- $v^2 = 20^2 + 2 \times 9.81 \times 30 = 988.6$, so $\mathbf{v = 31.4 \text{ m/s}}$.
- $v_y = 10 - 9.81 \times 3.694 = -26.2 \text{ m/s}$, so $\theta = \tan^{-1}(26.2 \div 17.32) = \mathbf{56.6^\circ}$.

Problem 18 Finding the launch angle

- $\sin 2\theta = gR \div u^2 = 9.81 \times 30 \div 400 = 0.7358$, so $2\theta = 47.4^\circ$ or 132.6° , giving $\theta = \mathbf{23.7^\circ}$ or $\mathbf{66.3^\circ}$.
- $T = 2u \sin \theta \div g$: $\mathbf{1.64 \text{ s}}$ for 23.7° and $\mathbf{3.73 \text{ s}}$ for 66.3° .

Problem 19 Motorbike jump

- $R = 22^2 \times \sin 50^\circ \div 9.81 = 484 \times 0.766 \div 9.81 = \mathbf{37.8 \text{ m}}$.
- 37.8 m is more than 35 m, so the rider **clears the gap**.
- $u^2 = gR \div \sin 2\theta = 343.4 \div 0.766 = 448.2$, so $\mathbf{u = 21.2 \text{ m/s}}$.

Problem 20 Launch from height and range

- $H = u^2 \sin^2 \theta \div 2g$ and $R = 2u^2 \sin \theta \cos \theta \div g$, so $H \div R = \tan \theta \div 4$ and $\mathbf{\tan \theta = 4H \div R}$.
- $\tan \theta = 72 \div 48 = 1.5$, so $\theta = \mathbf{56.3^\circ}$.
- $u^2 = gR \div \sin 2\theta = 470.9 \div 0.9231 = 510.1$, so $\mathbf{u = 22.6 \text{ m/s}}$.