

Radioactive decay and nuclear binding energy

Physics grades 11-12 (A Level · IB · Class 11-12) · Lesson

Quick answer: Radioactive decay is the random change of an unstable nucleus, and a sample decays exponentially with a fixed half-life. Nuclei are lighter than their separate nucleons, and $E = mc^2$ turns this mass defect into binding energy. Fission and fusion release energy because the products are more tightly bound per nucleon.

Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** IB E.3-E.5

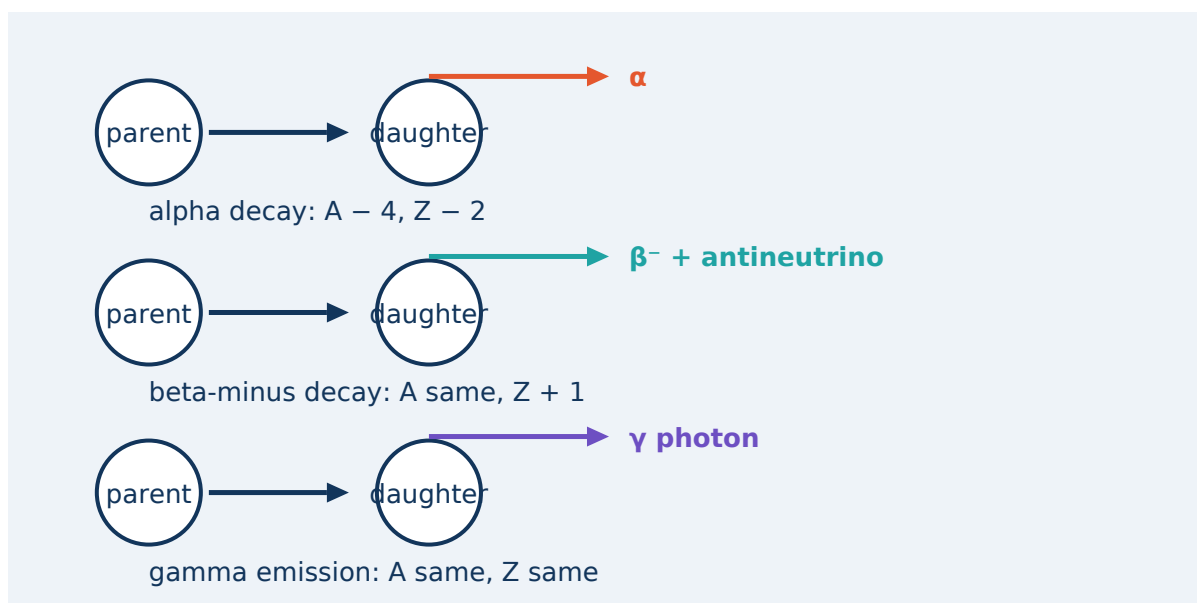
What happens in radioactive decay?

An unstable nucleus changes into a more stable one by emitting radiation. The process is random: nobody can say when one particular nucleus will decay, yet a large sample behaves very predictably. Temperature, pressure and chemical bonding have no effect, because the change happens deep inside the nucleus.

Definition

Radioactive decay is the spontaneous and random change of an unstable nucleus into another nucleus (the **daughter**) with emission of alpha, beta or gamma radiation. A nuclide is written with its nucleon number A (protons plus neutrons) and its proton number Z .

Decay	Emitted	Change in A	Change in Z	Example
alpha (α)	helium-4 nucleus	-4	-2	uranium-238 $\rightarrow$ thorium-234 + α
beta-minus (β^-)	fast electron and antineutrino	0	+1	carbon-14 $\rightarrow$ nitrogen-14 + e^- + antineutrino
gamma (γ)	high-energy photon	0	0	excited daughter nucleus settles down



Effect of each decay on the nucleus: only alpha decay changes the nucleon number.

Key law

In every nuclear equation the total nucleon number A and the total charge (proton number Z) are the same on both sides. Check the top numbers add up, then check the bottom numbers.

How does the decay law work?

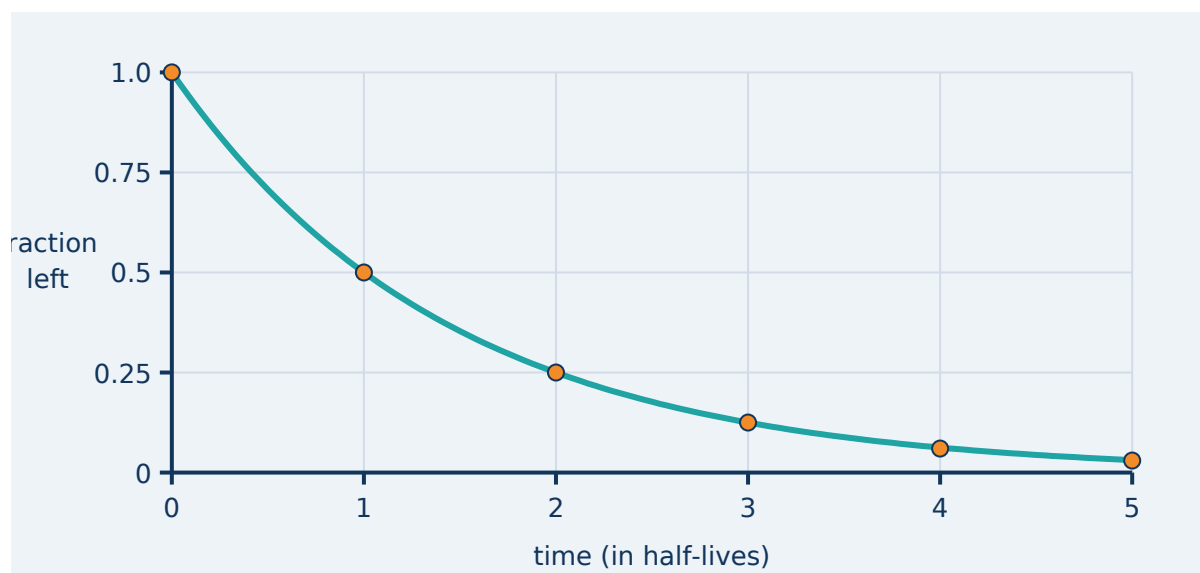
Every unstable nucleus has the same small chance of decaying in each second, called the decay constant λ . The number of decays per second, the **activity**, is therefore proportional to the number of undecayed nuclei, and the sample decays exponentially.

Formula

$$N = N_0 e^{-\lambda t}, A = A_0 e^{-\lambda t}, A = \lambda N, T_{1/2} = \ln 2 / \lambda \approx 0.693 / \lambda$$

N = undecayed nuclei, N_0 = initial number, A = activity (Bq, decays per second), λ = decay constant (s^{-1}), t = time (s), $T_{1/2}$ = half-life (s).

The half-life is the time for the activity, or the number of undecayed nuclei, to fall to half. After n half-lives a fraction $(\frac{1}{2})^n$ remains.



Each half-life halves what is left: 1, 1/2, 1/4, 1/8 and so on.

Worked example 1

Strontium-90 has a half-life of 28.8 years. What fraction of a sample remains after 100 years?

$$\lambda = 0.693 / 28.8 = 0.0241 \text{ y}^{-1}. \text{ Then } N / N_0 = e^{-\lambda t} = e^{-2.41} = 0.0901.$$

About **9.0 %** remains.

Common mistake

After two half-lives the sample has not gone: one quarter remains, not zero. The half-life tells you how fast the sample shrinks, never when it ends.

What is binding energy and how does $E = mc^2$ explain it?

A nucleus is lighter than the separate protons and neutrons that make it. The difference is the **mass defect** Δm , and the energy equivalent of that missing mass was released when the nucleus formed.

Definition

The **binding energy** of a nucleus is the minimum energy needed to separate it completely into free protons and neutrons. It equals the mass defect times c^2 .

Formula

$$E = \Delta m c^2, \text{ and with } 1 \text{ u} = 931.5 \text{ MeV}/c^2, \text{ } E \text{ (MeV)} = \Delta m \text{ (u)} \times 931.5$$

Δm = mass of separate nucleons – mass of the nucleus (u or kg), $c = 3.00 \times 10^8 \text{ m/s}$, $1 \text{ MeV} = 1.60 \times 10^{-13} \text{ J}$.

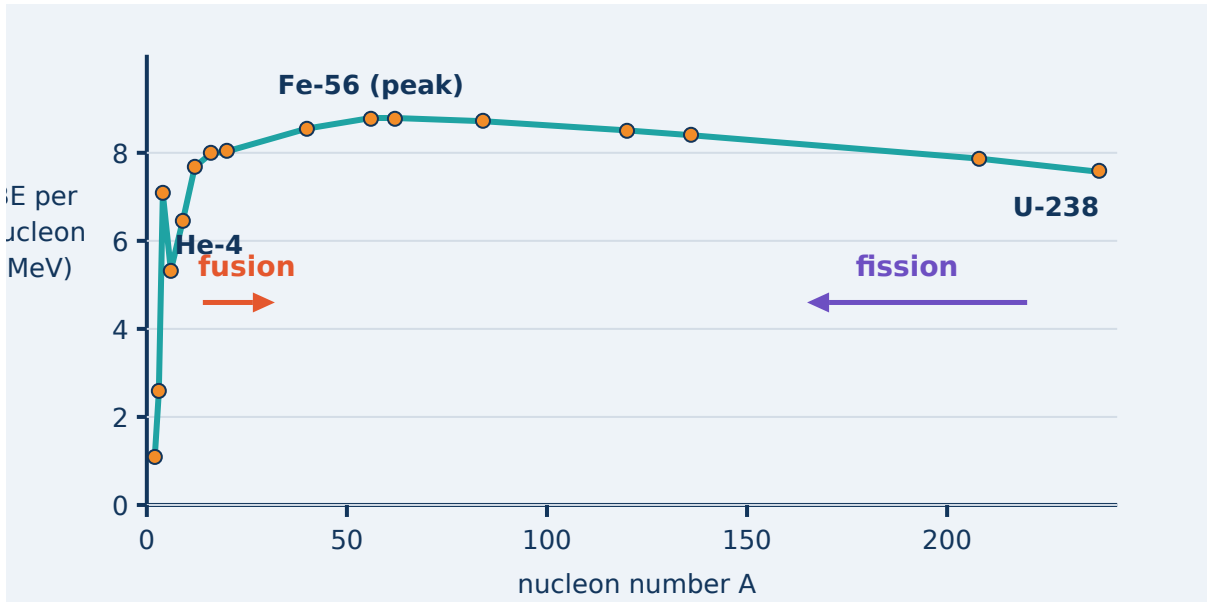
Worked example 2

Find the binding energy per nucleon of helium-4, using atomic masses: hydrogen atom

1.007825 u, neutron 1.008665 u, helium-4 atom 4.002603 u.

Mass of separate parts = $2 \times 1.007825 + 2 \times 1.008665 = 4.032980$ u, so $\Delta m = 0.030377$ u.

$E = 0.030377 \times 931.5 = 28.3$ MeV, and per nucleon $28.3 / 4 = \mathbf{7.07\text{ MeV}}$.

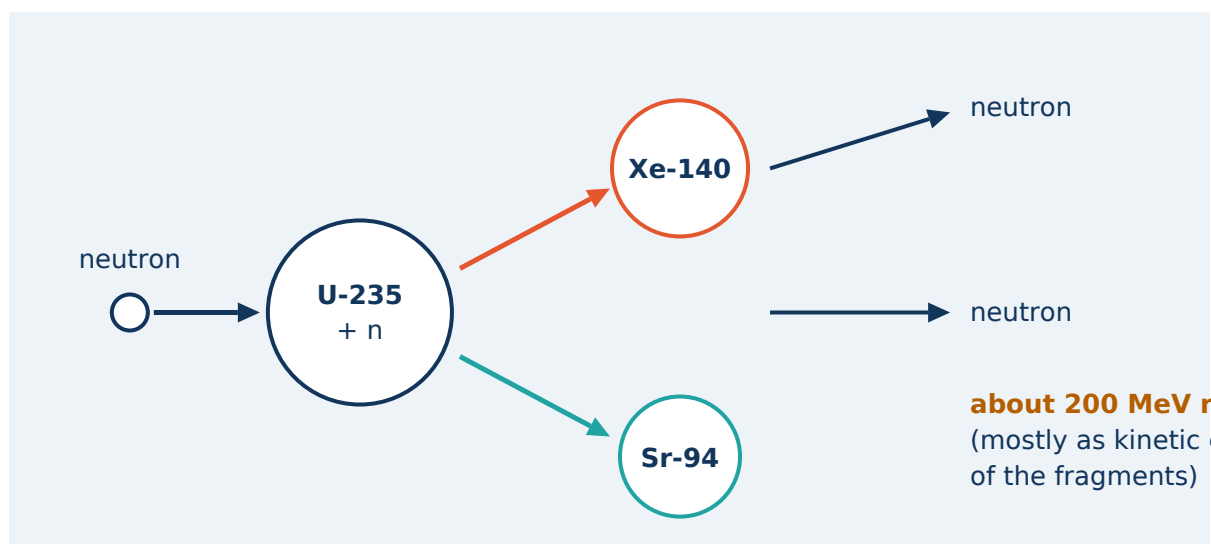


Binding energy per nucleon peaks near iron-56, so the most stable nuclei are in the middle of the table.

Why do fission and fusion release energy?

Energy is released whenever nuclei change into others with a higher binding energy per nucleon. Light nuclei can climb the curve by joining together (fusion). Very heavy nuclei can climb it by splitting (fission).

Feature	Fission	Fusion
Process	heavy nucleus splits into two medium ones	light nuclei join into a heavier one
Typical reaction	uranium-235 + n → xenon-140 + strontium-94 + 2n	deuterium + tritium → helium-4 + n
Energy released	about 200 MeV per nucleus	17.6 MeV per reaction
Condition	slow neutrons trigger a chain reaction	temperatures of 10 ⁷ to 10 ⁸ K to beat repulsion



A neutron splits uranium-235; the extra neutrons can split further nuclei, giving a chain reaction.

Lina's tip

For any nuclear equation, write A on top and Z at the bottom, then add each row separately. If a row does not balance, a particle is missing. In decay, ask which row changes: only Z changes in beta decay.

Key points to remember

- $N = N_0 e^{(-\lambda t)}$, $A = \lambda N$ and $T_{1/2} = \ln 2 / \lambda$
- After n half-lives a fraction $(\frac{1}{2})^n$ remains
- Binding energy $E = \Delta m c^2$, and $1 \text{ u} = 931.5 \text{ MeV}/c^2$
- Iron-56 sits near the peak of binding energy per nucleon, about 8.8 MeV
- Fission and fusion both move nuclei toward higher binding energy per nucleon

Radioactive decay and nuclear binding energy: frequently asked questions

What is half-life?

It is the time taken for the activity of a sample, or the number of undecayed nuclei, to fall to half its value. It is constant for a given nuclide and is not changed by heating, pressure or chemical state.

How are decay constant and half-life related?

$T_{1/2} = \ln 2 / \lambda$, which is about 0.693 divided by λ . A large λ means rapid decay and a short half-life.

What is the mass defect?

It is the amount by which a nucleus is lighter than its separate protons and neutrons. Multiplying it by c^2 gives the binding energy released when the nucleus formed.

Why is iron-56 the most stable nucleus?

It has about the highest binding energy per nucleon, near 8.8 MeV. Nuclei lighter or heavier than iron can release energy by moving toward it, by fusion or by fission.

How is nuclear fusion different from fission?

Fusion joins light nuclei into a heavier one, while fission splits a heavy nucleus into lighter ones. Both release energy because the products have a higher binding energy per nucleon.