

Radioactive decay and nuclear binding energy

Physics grades 11-12 (A Level · IB · Class 11-12) · Worksheet answers

Every answer shows the formula, the substitution and the result with its unit. In nuclear equations, check the top row (nucleon number) and the bottom row (proton number) separately. In decay problems, turn the time into a number of half-lives before you do anything else.

Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** IB E.3-E.5

Problem 1 Three kinds of radiation

1. An alpha particle is a helium-4 nucleus (two protons, two neutrons); A falls by 4 and Z by 2.
2. A neutron changes into a proton, with an electron and an antineutrino emitted; Z rises by 1 and A stays the same.
3. **A and Z do not change**; the nucleus only loses energy.

Problem 2 Radium alpha decay

A: $226 = A(\text{radon}) + 4$, so $A = 222$. Z: $88 = 86 + 2$, which balances. **Radium-226 → radon-222 + alpha particle (helium-4).**

Problem 3 Cobalt-60 beta decay

In beta-minus decay A is unchanged, so $A = 60$, and Z rises by 1, so $Z = 28$. **The daughter is nickel-60 (Z = 28).**

Problem 4 Three half-lives of cobalt-60

$15.8 / 5.27 = 3.00$ half-lives. Each half-life halves the activity: $8.0 \rightarrow 4.0 \rightarrow 2.0 \rightarrow 1.0$ ($\times 10^6$ Bq). **Activity = 1.0×10^6 Bq.**

Problem 5 Mass and energy units

1. $0.0305 \times 931.5 = \mathbf{28.4 \text{ MeV}}$.
2. $4.0 \times 1.60 \times 10^{-13} = \mathbf{6.4 \times 10^{-13} \text{ J}}$.

Problem 6 Decay constant of caesium-137

$T_{1/2} = 30.1 \times 3.156 \times 10^7 = 9.50 \times 10^8 \text{ s}$. $\lambda = 0.693 / T_{1/2} = 0.693 / 9.50 \times 10^8 = \mathbf{7.30 \times 10^{-10} \text{ s}^{-1}}$.

Problem 7 Activity of a radon sample

- $T_{1/2} = 3.82 \times 86\,400 = 330\,000 \text{ s}$, so $\lambda = 0.693 / 330\,000 = \mathbf{2.10 \times 10^{-6} \text{ s}^{-1}}$.
- $A = \lambda N = 2.10 \times 10^{-6} \times 2.0 \times 10^{15} = \mathbf{4.2 \times 10^9 \text{ Bq}}$.

Problem 8 Phosphorus-32 tracer

Number of half-lives = $30 / 14.3 = 2.098$. Fraction left = $(\frac{1}{2})^{2.098} = 0.234$. **23.4 %** remains.

Problem 9 Reading a decay graph

- The activity falls from 960 Bq to 480 Bq at 15 hours: **$T_{1/2} = 15 \text{ hours}$** .
- 45 h is three half-lives: $960 \rightarrow 480 \rightarrow 240 \rightarrow 120$, so **120 Bq**.
- 240 Bq is two half-lives, **30 hours**.

Problem 10 A PET tracer

$A = A_0 (\frac{1}{2})^n$, so $n = \log_2(2000 / 300) = 2.737$ half-lives. $t = 2.737 \times 110 = \mathbf{301 \text{ minutes}}$ (5.0 hours).

Problem 11 Binding energy of the deuteron

Mass of parts = $1.007825 + 1.008665 = 2.016490 \text{ u}$. $\Delta m = 2.016490 - 2.014102 = 0.002388 \text{ u}$. $E = 0.002388 \times 931.5 = \mathbf{2.22 \text{ MeV}}$; per nucleon **1.11 MeV**.

Problem 12 A milligram of mass

- $m = 1.0 \times 10^{-6} \text{ kg}$; $E = mc^2 = 1.0 \times 10^{-6} \times (3.00 \times 10^8)^2 = \mathbf{9.0 \times 10^{10} \text{ J}}$.
- $9.0 \times 10^{10} / 3.6 \times 10^6 = \mathbf{25000 \text{ kWh}}$.
- $25000 / 5.0 = 5000 \text{ days} = \mathbf{13.7 \text{ years}}$.

Problem 13 Energy of an alpha decay

$\Delta m = 226.02541 - (222.01758 + 4.00260) = 226.02541 - 226.02018 = 0.00523 \text{ u}$. $E = 0.00523 \times 931.5 = \mathbf{4.87 \text{ MeV}}$, shared as kinetic energy of the alpha particle and the radon nucleus.

Problem 14 Reading a decay series

- Step 1: A falls by 4 and Z by 2, so alpha decay. Steps 2 and 3: A stays, Z rises by 1, so beta-minus decay. Step 4: A falls by 4, Z by 2, so alpha decay.
- Neutrons = $A - Z = 238 - 92 = \mathbf{146}$.
- Both have $Z = 92$, so they are **isotopes of uranium** with different numbers of neutrons (146 and 142).

Problem 15 Spot the errors

1. Wrong: after two half-lives one quarter of the nuclei remain. 2. Wrong: the mass number is unchanged, because a neutron is replaced by a proton; the proton number rises by one. 3. Wrong: binding energy is the energy that must be *supplied* to pull the nucleus apart into separate nucleons. **One quarter remains; A unchanged; energy supplied.**

Problem 16 Reading a binding energy chart

1. **Iron-56**, because it has the largest binding energy per nucleon (8.79 MeV), so the most energy is needed per nucleon to take it apart.
 2. Total = $56 \times 8.79 = \mathbf{492.2 \text{ MeV}}$. $\Delta m = 492.2 / 931.5 = \mathbf{0.528 \text{ u}}$.
 3. Binding energy of helium-4 = $4 \times 7.07 = 28.28 \text{ MeV}$; of two hydrogen-2 = $2 \times 2 \times 1.11 = 4.44 \text{ MeV}$. Energy released = $28.28 - 4.44 = \mathbf{23.8 \text{ MeV}}$.

Problem 17 Fission of uranium-235

1. A: $235 + 1 = 140 + 94 + x$ gives $x = 2$; Z: $92 = 54 + 38$ balances. **Two neutrons**.
 2. Products: $140 \times 8.29 + 94 \times 8.59 = 1160.6 + 807.46 = 1968.1 \text{ MeV}$. Reactant: $235 \times 7.59 = 1783.6 \text{ MeV}$. Energy released = $1968.1 - 1783.6 = \mathbf{184 \text{ MeV}}$.
 3. $N = (1000 / 235) \times 6.02 \times 10^{23} = 2.56 \times 10^{24}$ nuclei. $E = 2.56 \times 10^{24} \times 184 \times 1.60 \times 10^{-13} = \mathbf{7.54 \times 10^{13} \text{ J}}$.

Problem 18 Fusion fuel per kilogram

1. Binding energy of helium-4 = $4 \times 7.07 = 28.28 \text{ MeV}$. Of H-2 = $2 \times 1.11 = 2.22 \text{ MeV}$ and of H-3 = $3 \times 2.83 = 8.49 \text{ MeV}$, total 10.71 MeV . Energy released = $28.28 - 10.71 = \mathbf{17.6 \text{ MeV}}$.
 2. Mass of pair = $5.00 \times 1.661 \times 10^{-27} = 8.30 \times 10^{-27} \text{ kg}$. Energy = $17.6 \times 1.60 \times 10^{-13} / 8.30 \times 10^{-27} = \mathbf{3.39 \times 10^{14} \text{ J per kg}}$.
 3. Ratio = $3.39 \times 10^{14} / 8 \times 10^{13} = 4.2$, so fusion fuel gives **about 4.2 times more energy per kilogram**.

Problem 19 A straight line from a decay

1. $A = A_0 e^{-\lambda t}$, so $\ln A = \ln A_0 - \lambda t$, which has the form $y = c + mx$ with gradient $-\lambda$.
 2. $\ln 80.0 = 4.382$; $\ln 31.8 = 3.459$. Gradient = $(3.459 - 4.382) / 8 = \mathbf{-0.115 \text{ h}^{-1}}$.
 3. $\lambda = \mathbf{0.115 \text{ h}^{-1}}$; $T_{1/2} = 0.693 / 0.115 = \mathbf{6.0 \text{ hours}}$.

Problem 20 A gram of radium

1. $N = (1.00 / 226) \times 6.02 \times 10^{23} = \mathbf{2.66 \times 10^{21}}$.
 2. $\lambda = 0.693 / (1600 \times 3.156 \times 10^7) = 1.37 \times 10^{-11} \text{ s}^{-1}$. $A = \lambda N = 1.37 \times 10^{-11} \times 2.66 \times 10^{21} = \mathbf{3.66 \times 10^{10} \text{ Bq}}$.
 3. $400 / 1600 = 0.25$ half-lives, so $A = 3.66 \times 10^{10} \times 2^{-0.25} = \mathbf{3.07 \times 10^{10} \text{ Bq}}$.