

Kirchhoff's rules and RC transients

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet

Name: _____ Date: _____ Class: _____

These twenty problems develop two linked skills: using Kirchhoff's junction and loop rules to analyse multi-branch circuits, and using exponentials to follow a capacitor as it charges and discharges. They move from simple recall and one-step calculations to derivations, graph analysis and energy arguments in the style of AP Physics C: Electricity and Magnetism. The set suits AP students, first-year university physics and A Level learners ready for calculus.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:E&M Unit 11

Problem 1 Junction rule (easy)

At a junction 3.0 A and 2.0 A flow in, and 4.0 A flows out along one wire.

1. State the junction rule.
2. Calculate the current in the remaining wire and say whether it enters or leaves.

Problem 2 Time constant (easy)

1. Calculate τ for $R = 10 \text{ k}\Omega$ and $C = 22 \text{ }\mu\text{F}$.
2. Show that the units of RC are seconds.

Problem 3 Start and finish (easy)

Aarav charges a $100 \text{ }\mu\text{F}$ capacitor through $1.5 \text{ k}\Omega$ from a 6.0 V battery.

1. Calculate the current at $t = 0$.
2. State the current as $t \rightarrow \infty$.
3. Calculate the final charge.

Problem 4 Fractions charged (easy)

1. State the fraction of the final charge present after $t = \tau$.
2. Calculate the fraction after 3τ .
3. Calculate the fraction after 5τ .

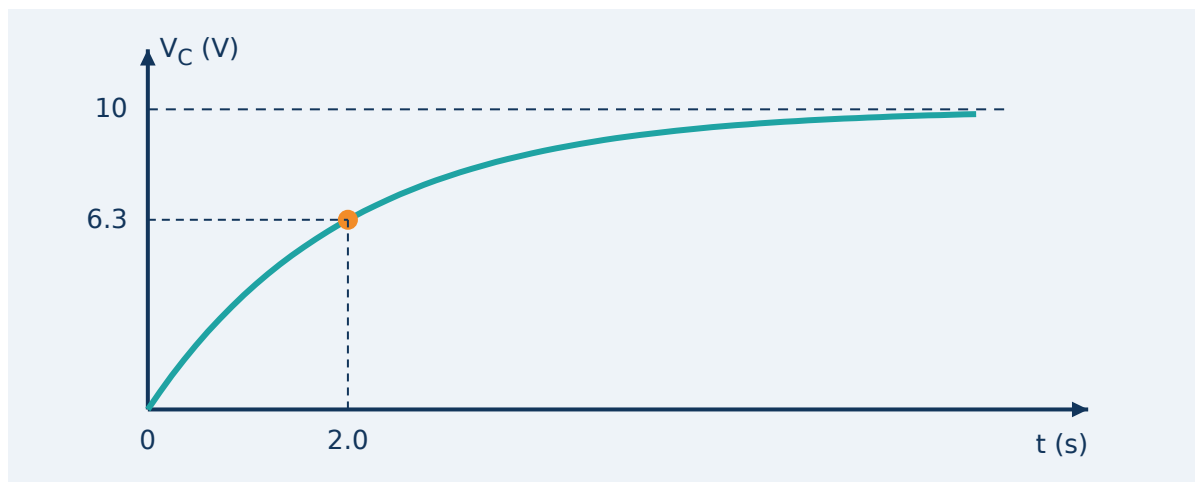
Problem 5 Discharge fraction (easy)

A capacitor with $Q_0 = 80 \text{ }\mu\text{C}$ discharges through a resistor.

1. Calculate the charge remaining after 2τ .
2. State what fraction has gone.

Problem 6 Reading a charging graph (easy)

Chiamaka records V_C against time while charging through a resistor.



Capacitor voltage while charging.

1. Read the time constant from the graph.
2. Calculate R if $C = 1000 \mu\text{F}$.

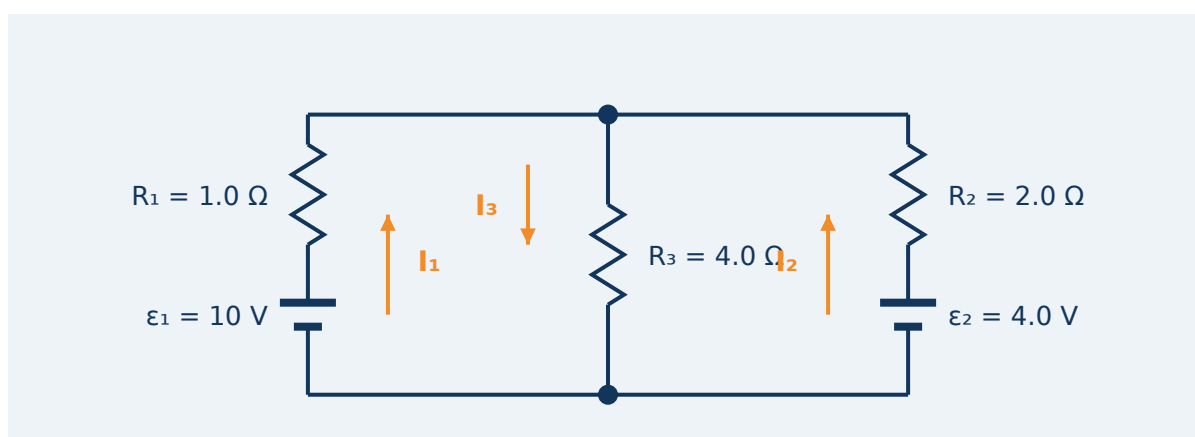
Problem 7 Resistors in a loop (easy)

A 12 V battery drives current through 4.0Ω and 2.0Ω in series.

1. Use the loop rule to calculate the current.
2. Calculate the voltage across the 4.0Ω resistor.

Problem 8 Two-loop circuit (medium)

Apply Kirchhoff's rules to the circuit shown.



Three branches between two junctions.

1. Write the junction equation.
2. Write the two loop equations.
3. Calculate I_1 , I_2 and I_3 , and say which battery is charging.
4. Check by comparing power supplied with power dissipated.

Problem 9 Steady state (medium)

A 12 V battery is in series with a 4.0 k Ω resistor, and then a 2.0 k Ω resistor in parallel with a 5.0 μ F capacitor.

1. Explain why no current flows in the capacitor branch after a long time.
2. Calculate the steady current.
3. Calculate the final voltage and charge on the capacitor.

Problem 10 Time to reach 8 V (medium)

Emma charges a 3.3 μ F capacitor through 2.0 M Ω from 12 V.

1. Calculate τ .
2. Calculate the time for V_c to reach 8.0 V.

Problem 11 Discharge current (medium)

A 2200 μ F capacitor at 9.0 V discharges through 470 Ω .

1. Calculate τ .
2. Calculate the initial current.
3. Calculate the current after 2.0 s.

Problem 12 Half-time (medium)

A 10 μ F capacitor discharges through 100 k Ω .

1. Show that the half-time is $\tau \ln 2$.
2. Calculate it for this circuit.
3. Calculate the time for the charge to fall to one sixteenth.

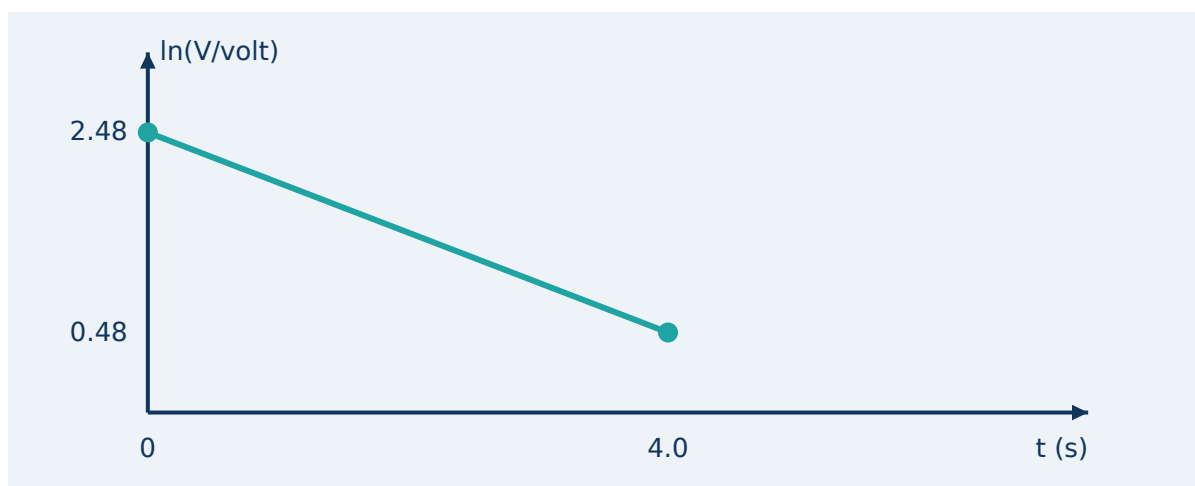
Problem 13 Verify the charging law (medium)

Take $q = C\varepsilon(1 - e^{-t/RC})$.

1. Differentiate to find I .
2. Show that the loop equation $\varepsilon = IR + q/C$ is satisfied.

Problem 14 Log graph of discharge (medium)

Bilal plots $\ln V$ against t for a discharging capacitor. The initial voltage is 12 V.



ln V against t.

1. Calculate the gradient.
2. Hence find τ .
3. Calculate R if $C = 470 \mu\text{F}$.

Problem 15 Battery with internal resistance (medium)

A 12 V battery of internal resistance 0.50Ω drives a 5.5Ω load.

1. Calculate the current.
2. Calculate the terminal voltage.

Problem 16 Energy in charging (hard)

A $100 \mu\text{F}$ capacitor is charged from 12 V through a resistor R.

1. Calculate the energy stored.
2. Integrate $I^2 R$ to show the resistor dissipates the same energy.
3. Explain why the answer does not depend on R.

Problem 17 Divider charging (hard)

A 12 V source feeds a $10 \text{ k}\Omega$ resistor to a node, where a $10 \mu\text{F}$ capacitor is in parallel with a $20 \text{ k}\Omega$ resistor to ground.

1. Calculate the final capacitor voltage.
2. Calculate the time constant using the equivalent resistance.
3. Calculate the time for the voltage to reach 6.0 V.

Problem 18 Leaky capacitor (hard)

A parallel-plate capacitor is filled with a material of dielectric constant 4.0 and resistivity $1.0 \times 10^{12} \Omega \cdot \text{m}$.

1. Show that the leakage time constant is $\rho k \epsilon_0$ whatever the plate size.
2. Calculate it ($\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$).

Problem 19 Flash lamp (hard)

Kofi's flash circuit has a $470\ \mu\text{F}$ capacitor charged to $25\ \text{V}$ that discharges through a $100\ \Omega$ lamp.

1. Calculate the initial energy.
2. Show that the energy dissipated up to time t is $U_0(1 - e^{-2t/\tau})$.
3. Calculate the energy dissipated in the first τ .

Problem 20 Spot the error (hard)

A student says: 'The capacitor reaches 63 % of full charge in τ , so it will be full after 1.6τ .'

The circuit has $\mathcal{E} = 6.0\ \text{V}$, $R = 10\ \text{k}\Omega$ and $\tau = 0.22\ \text{s}$.

1. Explain the flaw.
2. Calculate the time to reach 99 %.
3. Calculate the current at that moment.