

Kirchhoff's rules and RC transients

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet answers

Complete methods follow: the equation, the substitution and the final result with its unit in bold. When a current comes out negative, note what that means physically, and when a time constant is involved, check that your answer is a plausible multiple of τ .

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:E&M Unit 11

Problem 1 Junction rule

1. The sum of currents entering a junction equals the sum leaving it.
2. $I_{\text{in}} = 5.0 \text{ A}$ and out so far $= 4.0 \text{ A}$, so **1.0 A leaves** along the remaining wire.

Problem 2 Time constant

1. $\tau = RC = (1.0 \times 10^4)(22 \times 10^{-6}) = \mathbf{0.22 \text{ s}}$.
2. $\Omega \times F = (V/A)(C/V) = C/A = \mathbf{s}$.

Problem 3 Start and finish

1. $I_0 = \mathcal{E}/R = 6.0/1500 = \mathbf{4.0 \text{ mA}}$.
2. The capacitor is an open circuit when full, so $I \rightarrow \mathbf{0}$.
3. $Q = C\mathcal{E} = (100 \times 10^{-6})(6.0) = \mathbf{6.0 \times 10^{-4} \text{ C}}$.

Problem 4 Fractions charged

1. $1 - e^{-1} = \mathbf{0.632}$.
2. $1 - e^{-3} = \mathbf{0.950}$.
3. $1 - e^{-5} = \mathbf{0.993}$.

Problem 5 Discharge fraction

1. $q = Q_0 e^{-2} = 80(0.1353) = \mathbf{10.8 \mu\text{C}}$.
2. **86.5 %** has gone.

Problem 6 Reading a charging graph

1. $V_C = 0.63 \times 10 \text{ V} = 6.3 \text{ V}$ at 2.0 s , so $\tau = \mathbf{2.0 \text{ s}}$.
2. $R = \tau/C = 2.0/(1000 \times 10^{-6}) = \mathbf{2.0 \text{ k}\Omega}$.

Problem 7 Resistors in a loop

- $12 - I(4.0 + 2.0) = 0$ gives $I = \mathbf{2.0\ A}$.
- $V = IR = 2.0 \times 4.0 = \mathbf{8.0\ V}$.

Problem 8 Two-loop circuit

- $I_3 = I_1 + I_2$.
- Left loop: $10 - 1.0I_1 - 4.0I_3 = 0$ gives $10 = 5I_1 + 4I_2$. Right loop: $4.0 - 2.0I_2 - 4.0I_3 = 0$ gives $4 = 4I_1 + 6I_2$.
- Eliminating I_2 : $22 = 7I_1$, so $I_1 = \mathbf{3.1\ A}$, $I_2 = (10 - 5 \times 3.143)/4 = \mathbf{-1.4\ A}$ and $I_3 = \mathbf{1.7\ A}$. Battery 2 is being charged.
- Supplied: $10(3.143) + 4(-1.429) = 25.7\ \text{W}$. Dissipated: $3.143^2 \times 1 + 1.429^2 \times 2 + 1.714^2 \times 4 = 9.88 + 4.08 + 11.76 = 25.7\ \text{W}$, which **agree**.

Problem 9 Steady state

- A full capacitor has no further voltage change, so charge stops flowing in its branch.
- $I = 12/(4000 + 2000) = \mathbf{2.0\ mA}$.
- $V_C = IR_2 = 4.0\ \text{V}$ and $Q = CV = \mathbf{20\ \mu C}$.

Problem 10 Time to reach 8 V

- $\tau = (2.0 \times 10^6)(3.3 \times 10^{-6}) = \mathbf{6.6\ s}$.
- $8 = 12(1 - e^{-t/\tau})$ gives $e^{-t/\tau} = 1/3$, so $t = \tau \ln 3 = \mathbf{7.3\ s}$.

Problem 11 Discharge current

- $\tau = (470)(2200 \times 10^{-6}) = \mathbf{1.03\ s}$.
- $I_0 = 9.0/470 = \mathbf{19\ mA}$.
- $I = 19.15\ \text{mA} \times e^{-2.0/1.034} = 19.15 \times 0.1445 = \mathbf{2.8\ mA}$.

Problem 12 Half-time

- Set $q = Q_0/2$: $e^{-t/\tau} = 1/2$, so $t_{1/2} = \tau \ln 2$.
- $\tau = (1.0 \times 10^5)(1.0 \times 10^{-5}) = 1.0\ \text{s}$, so $t_{1/2} = \mathbf{0.69\ s}$.
- $1/16 = (1/2)^4$ so four half-times: $\mathbf{2.8\ s}$.

Problem 13 Verify the charging law

- $I = dq/dt = C\varepsilon(1/RC)e^{-t/RC} = (\varepsilon/R)e^{-t/RC}$.
- $IR + q/C = \varepsilon e^{-t/RC} + \varepsilon(1 - e^{-t/RC}) = \varepsilon$, so the loop equation holds.

Problem 14 Log graph of discharge

- Gradient = $(0.48 - 2.48)/4.0 = \mathbf{-0.50\ s^{-1}}$.

2. $\ln V = \ln V_0 - t/\tau$, so $\tau = 1/0.50 = \mathbf{2.0\ s}$.
3. $R = \tau/C = 2.0/(470 \times 10^{-6}) = \mathbf{4.3\ k\Omega}$.

Problem 15 Battery with internal resistance

1. $I = \mathcal{E}/(R + r) = 12/6.0 = \mathbf{2.0\ A}$.
2. $V = \mathcal{E} - Ir = 12 - 1.0 = \mathbf{11\ V}$.

Problem 16 Energy in charging

1. $U = \frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}(100 \times 10^{-6})(144) = \mathbf{7.2 \times 10^{-3}\ J}$.
2. $\int I^2 R\ dt = (\mathcal{E}^2/R) \int_0^\infty e^{-2t/RC} dt = (\mathcal{E}^2/R)(RC/2) = \frac{1}{2}C\mathcal{E}^2 = 7.2\ \text{mJ}$.
3. R cancels: a larger R slows the current but lengthens the time, leaving the energy unchanged.

Problem 17 Divider charging

1. $V_{\text{final}} = 12 \times 20/(10 + 20) = \mathbf{8.0\ V}$.
2. $R_{\text{eq}} = 10 \parallel 20 = 6.67\ \text{k}\Omega$, so $\tau = (6.67 \times 10^3)(10 \times 10^{-6}) = \mathbf{0.067\ s}$.
3. $6.0 = 8.0(1 - e^{-t/\tau})$ gives $e^{-t/\tau} = 0.25$, so $t = \tau \ln 4 = \mathbf{0.092\ s}$.

Problem 18 Leaky capacitor

1. $R = \rho d/A$ and $C = \kappa \epsilon_0 A/d$, so $RC = \rho \kappa \epsilon_0$: A and d cancel.
2. $\tau = (1.0 \times 10^{12})(4.0)(8.85 \times 10^{-12}) = \mathbf{35\ s}$.

Problem 19 Flash lamp

1. $U_0 = \frac{1}{2}(470 \times 10^{-6})(625) = \mathbf{0.15\ J}$.
2. $P = V^2/R = (V_0^2/R)e^{-2t/\tau}$; integrating from 0 to t gives $(V_0^2\tau/2R)(1 - e^{-2t/\tau})$, and $V_0^2\tau/2R = \frac{1}{2}CV_0^2 = U_0$.
3. $U_0(1 - e^{-2}) = 0.1469(0.8647) = \mathbf{0.13\ J}$.

Problem 20 Spot the error

1. Each τ removes the same fraction (63 %) of what is left, so charging never completes in a fixed finite time.
2. $1 - e^{-t/\tau} = 0.99$ gives $t = \tau \ln 100 = 0.22 \times 4.605 = \mathbf{1.0\ s}$.
3. $I = (6.0/10^4)(0.01) = \mathbf{6.0\ \mu A}$.