

Rotational inertia and Newton's second law

AP Physics 1 (algebra-based) · Lesson

Quick answer: Rotational inertia I measures how strongly an object resists changes in its rotation and depends on both mass and how far that mass is from the axis. Newton's second law for rotation states that net torque equals rotational inertia times angular acceleration, $\tau_{\text{net}} = I\alpha$, with I in kg m^2 .

Level: AP Physics 1 (algebra-based) · **Curriculum:** AP P1 Unit 5

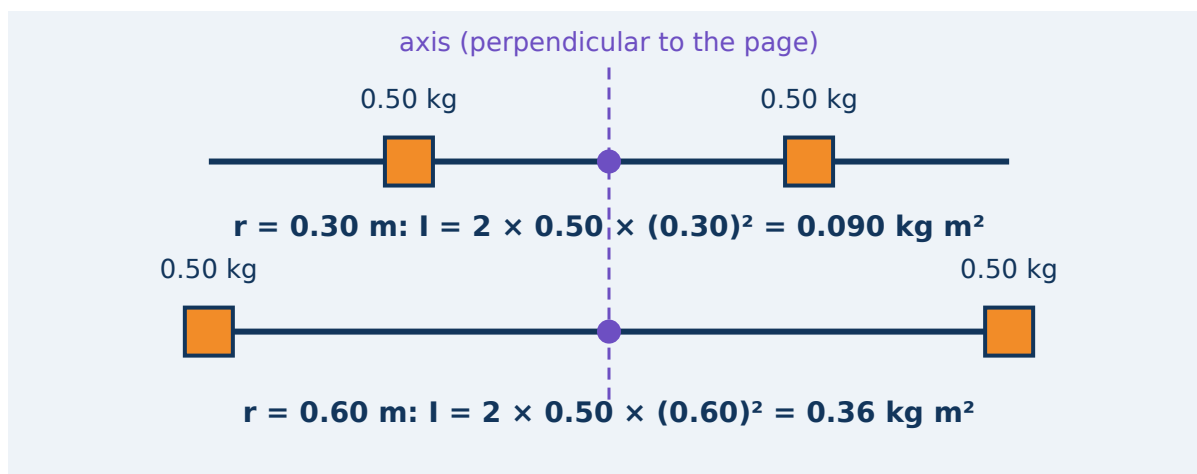
What is rotational inertia?

Pushing a heavy door near its hinges is much harder than pushing it at the handle, and a long pole is harder to spin than a short one of equal mass. In rotation, mass alone does not decide how hard it is to change the motion; the distance of the mass from the axis matters even more.

Definition

Rotational inertia (moment of inertia) I measures an object's resistance to changes in its rotation about a chosen axis. For a collection of point masses, $I = \sum mr^2$, where r is each mass's distance from the axis. Its SI unit is kg m^2 .

Because r is squared, doubling the distance of a mass from the axis multiplies its contribution by four. Figure 1 shows the same two masses at two different distances.



Moving the same masses twice as far from the axis makes the rotational inertia four times larger.

Worked example 1

Two 0.50 kg masses sit on a light rod, with the axis through its centre. Find I when each mass is 0.30 m from the axis, then 0.60 m from it.

Method: $I = \sum mr^2$. At 0.30 m: $I = 2 \times 0.50 \times (0.30)^2 = 0.090 \text{ kg m}^2$. At 0.60 m: $I = 2 \times 0.50 \times (0.60)^2 = \mathbf{0.36 \text{ kg m}^2}$. Doubling r multiplied I by 4.

Real objects have continuous mass, so their values come from calculus and are listed in tables. The value always depends on the axis chosen.

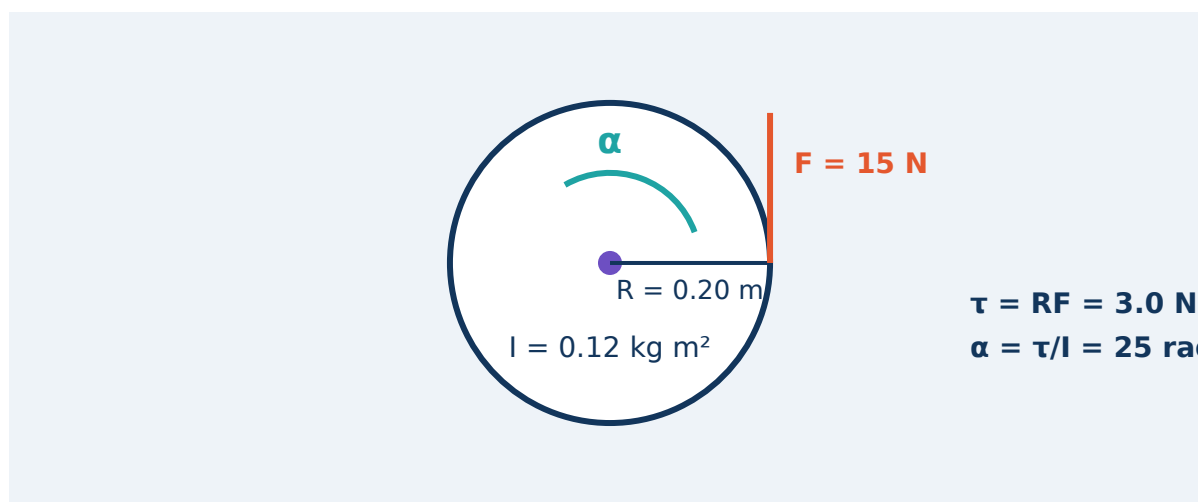
Object	Axis	Rotational inertia
Hoop or thin ring, mass M , radius R	through the centre, perpendicular to the plane	MR^2
Solid disk or cylinder	through the centre	$\frac{1}{2}MR^2$
Solid sphere	through the centre	$(2/5)MR^2$
Uniform rod, length L	through the centre, perpendicular to the rod	$(1/12)ML^2$
Uniform rod, length L	through one end	$(1/3)ML^2$

What is Newton's second law for rotation?

A net force gives a straight-line object an acceleration $a = F/m$. The rotational partner uses net torque, the turning effect of forces, and rotational inertia in place of mass. The torque of a force F at distance r from the axis is $\tau = rF \sin\theta$, where θ is the angle between the force and the line from the axis.

Key law

$\tau_{\text{net}} = I\alpha$. The net torque about an axis equals the rotational inertia about that axis multiplied by the angular acceleration (rad/s^2). The torque and α share a sign: positive for counterclockwise, negative for clockwise, once you have chosen a direction.



The tangential force gives a torque; dividing by the rotational inertia gives the angular acceleration.

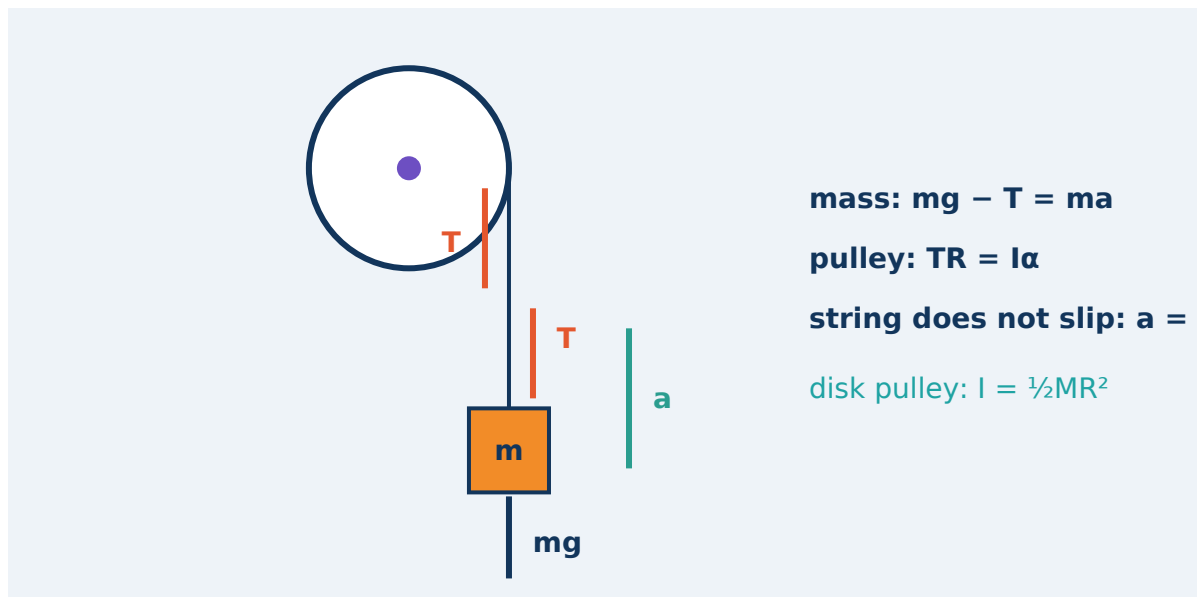
Worked example 2

A tangential 15 N force acts on the rim of the wheel in Figure 2 ($R = 0.20 \text{ m}$, $I = 0.12 \text{ kg m}^2$). Find the angular acceleration and the angular speed after 3.0 s from rest.

Method: $\tau = RF = 0.20 \times 15 = 3.0 \text{ N m}$. $\alpha = \tau/I = 3.0 \div 0.12 = 25 \text{ rad/s}^2$. Then $\omega = \alpha t = 25 \times 3.0 = \mathbf{75 \text{ rad/s}}$.

How do you combine rotation with straight-line motion?

When a string winds around a pulley, the hanging mass moves in a straight line while the pulley spins. Apply Newton's second law twice and link the two with $a = R\alpha$, which holds if the string does not slip. Figure 3 shows the standard set-up.



Two applications of Newton's second law, one for the straight-line motion and one for the spinning disk, linked by $a = R\alpha$.

Worked example 3

A 2.0 kg mass hangs from a string wound on a solid disk pulley of mass 4.0 kg and radius 0.10 m. Find the acceleration and the tension.

Method: $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 4.0 \times (0.10)^2 = 0.020 \text{ kg m}^2$. Substituting $\alpha = a/R$ in $TR = I\alpha$ gives $T = Ia/R^2 = \frac{1}{2}Ma = 2.0a$. Then $mg - T = ma$ gives $2.0 \times 9.81 - 2.0a = 2.0a$, so **$a = 4.9 \text{ m/s}^2$** (4.905), and $T = 2.0 \times 4.905 = \mathbf{9.8 \text{ N}}$. The tension is only half of $mg = 19.6 \text{ N}$ because the mass is accelerating.

Common mistake

Treating rotational inertia as a fixed property of the object. It depends on the chosen axis: a rod has $I = (1/12)ML^2$ about its centre but $(1/3)ML^2$ about one end, four times as much. Also, do not assume the string tension equals mg when the hanging mass accelerates.

Lina's tip

Rotation problems are Newton's second law twice: one equation for the straight-line motion and one for the spinning part. Link them with $a = R\alpha$, and check that a and α point the same way before you substitute.

Key points to remember

- Rotational inertia of point masses: $I = \sum mr^2$, in kg m^2

- Newton's second law for rotation: $\tau_{\text{net}} = I\alpha$
- Torque: $\tau = rF \sin\theta$, in N m
- Solid disk: $I = \frac{1}{2}MR^2$; hoop: $I = MR^2$; rod about its centre: $I = (1/12)ML^2$
- A string that does not slip links the motions: $a = R\alpha$

Rotational inertia and Newton's second law: frequently asked questions

What is the difference between mass and rotational inertia?

Mass resists changes in straight-line motion and does not depend on where you push. Rotational inertia resists changes in rotation and depends on the axis and on how the mass is distributed. The same object can have different values of I about different axes.

Why does moving mass outwards increase rotational inertia so much?

Each mass contributes mr^2 , so the distance is squared. Doubling the distance multiplies that mass's contribution by four. This is why a long pole is harder to turn than a compact object of the same mass.

How is Newton's second law for rotation related to $F = ma$?

They have the same structure. Net force becomes net torque, mass becomes rotational inertia and acceleration becomes angular acceleration, giving $\tau_{\text{net}} = I\alpha$. Both say that a bigger cause or a smaller resistance gives a bigger change in motion.

What does the sign of torque mean?

Torque is a vector along the axis. In a plane, counterclockwise is usually taken as positive and clockwise as negative. Torques with the same sense add, those with opposite senses subtract, and the sign of the net torque gives the sense of α .

When can I use $a = R\alpha$?

Use it when a string or belt moves without slipping around a pulley or wheel of radius R . Then the point on the rim has the same tangential speed and tangential acceleration as the string, so the linear acceleration a equals $R\alpha$.