

Rotational inertia and Newton's second law

AP Physics 1 (algebra-based) · Worksheet answers

Each answer shows the law, the substitution and the final value in bold. When a problem has several steps, check that your signs and axis choice are consistent, and compare your units with the expected ones: kg m^2 , N m or rad/s^2 .

Level: AP Physics 1 (algebra-based) · **Curriculum:** AP P1 Unit 5

Problem 1 Units and symbols

1. **kg m^2** .
2. **N m** .
3. **rad/s^2** .
4. Rotational inertia measures how strongly an object resists a change in its angular velocity about a given axis, and it depends on mass and on how that mass is distributed relative to the axis.

Problem 2 A single point mass

1. $I = mr^2 = 0.40 \times (0.50)^2 = \mathbf{0.10 \text{ kg m}^2}$.
2. $I = 0.40 \times (1.0)^2 = \mathbf{0.40 \text{ kg m}^2}$, which is **4 times** larger since r doubled.

Problem 3 Torque from a force

1. $\tau = rF = 0.40 \times 25 = \mathbf{10 \text{ N m}}$.
2. $\tau = rF \sin\theta = 0.40 \times 25 \times \sin 30^\circ = \mathbf{5.0 \text{ N m}}$.

Problem 4 Angular acceleration

1. $\alpha = \tau/I = 6.0 \div 0.30 = \mathbf{20 \text{ rad/s}^2}$.
2. $t = \omega/\alpha = 40 \div 20 = \mathbf{2.0 \text{ s}}$.

Problem 5 Finding rotational inertia

1. $I = \tau/\alpha = 12 \div 4.0 = \mathbf{3.0 \text{ kg m}^2}$.
2. With $I = 6.0 \text{ kg m}^2$: $\alpha = 12 \div 6.0 = \mathbf{2.0 \text{ rad/s}^2}$, half as large.

Problem 6 Hoop and disk

1. Hoop: $I = MR^2 = 2.0 \times (0.30)^2 = \mathbf{0.18 \text{ kg m}^2}$. Disk: $I = \frac{1}{2}MR^2 = \mathbf{0.090 \text{ kg m}^2}$.

2. The disk is easier to spin up: it has half the rotational inertia, because more of its mass is close to the axis, so the same torque gives twice the angular acceleration.

Problem 7 Dumbbell axes

1. Each mass is 0.50 m from the centre: $I = 2 \times 0.80 \times (0.50)^2 = \mathbf{0.40 \text{ kg m}^2}$.
2. One mass is on the axis ($r = 0$) and the other is 1.0 m away: $I = 0.80 \times (1.0)^2 = \mathbf{0.80 \text{ kg m}^2}$.
3. The end axis gives twice the value, because the moving mass is twice as far out and r^2 makes it count four times, while only one mass contributes instead of two.

Problem 8 Masses at the corners of a square

1. The distance from the centre to a corner is half the diagonal: $0.60\sqrt{2} \div 2 = 0.424 \text{ m}$, so $r^2 = 0.18 \text{ m}^2$. $I = 4 \times 0.50 \times 0.18 = \mathbf{0.36 \text{ kg m}^2}$.
2. From a corner: distances² are 0, 0.36, 0.36 and 0.72 m^2 . $I = 0.50 \times (0 + 0.36 + 0.36 + 0.72) = \mathbf{0.72 \text{ kg m}^2}$.
3. Axis A (the centre), because its rotational inertia is half as large.

Problem 9 Pulling a rope on a wheel

1. $\tau = RF = 0.25 \times 18 = \mathbf{4.5 \text{ N m}}$.
2. $\alpha = \tau/I = 4.5 \div 0.50 = \mathbf{9.0 \text{ rad/s}^2}$.
3. $\omega = \alpha t = 9.0 \times 3.0 = \mathbf{27 \text{ rad/s}}$.

Problem 10 Slowing a flywheel

1. $\alpha = (0 - 60) \div 12 = \mathbf{-5.0 \text{ rad/s}^2}$.
2. $\tau = I\alpha = 0.80 \times (-5.0) = \mathbf{-4.0 \text{ N m}}$.
3. $F = \tau/r = 4.0 \div 0.10 = \mathbf{40 \text{ N}}$.

Problem 11 Solid disk

1. $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 3.0 \times (0.20)^2 = \mathbf{0.060 \text{ kg m}^2}$.
2. $\alpha = \tau/I = 0.30 \div 0.060 = \mathbf{5.0 \text{ rad/s}^2}$.

Problem 12 Opening a door

1. $\tau = rF = 0.90 \times 20 = \mathbf{18 \text{ N m}}$.
2. $\alpha = 18 \div 4.0 = \mathbf{4.5 \text{ rad/s}^2}$.
3. $\theta = \frac{1}{2}\alpha t^2$, so $t = \sqrt{(2\theta/\alpha)} = \sqrt{(2 \times 1.571 \div 4.5)} = \sqrt{0.698} = \mathbf{0.84 \text{ s}}$.

Problem 13 Choking up on a bat

1. Sliding the hands towards the middle puts the axis nearer the centre of mass, so less of the bat's mass is far from the axis. Since $I = \sum mr^2$, the rotational inertia drops sharply.
2. With $\tau = I\alpha$, a smaller I gives a **larger angular acceleration** for the same torque from the hands.

Problem 14 Pulley and a hanging mass

1. $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 2.0 \times (0.050)^2 = \mathbf{2.5 \times 10^{-3} \text{ kg m}^2}$.
2. With $T = \frac{1}{2}Ma = 1.0a$ and $mg - T = ma$: $1.5 \times 9.81 = (1.5 + 1.0)a$, so $a = 14.715 \div 2.5 = \mathbf{5.9 \text{ m/s}^2}$.
3. $T = 1.0 \times 5.886 = \mathbf{5.9 \text{ N}}$; $\alpha = a/R = 5.886 \div 0.050 = \mathbf{1.2 \times 10^2 \text{ rad/s}^2}$ (118 rad/s²).

Problem 15 Two masses and a pulley

1. $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 2.0 \times 0.010 = 0.010 \text{ kg m}^2$. The 3.0 kg mass moves down. Net force along the string: $(m_1 - m_2)g = (m_1 + m_2 + \frac{1}{2}M)a$, so $a = 2.0 \times 9.81 \div 5.0 = \mathbf{3.9 \text{ m/s}^2}$ (3.924).
2. $T_1 = m_1(g - a) = 3.0 \times 5.886 = \mathbf{18 \text{ N}}$ (17.7 N). $T_2 = m_2(g + a) = 1.0 \times 13.734 = \mathbf{14 \text{ N}}$ (13.7 N).
3. $(T_1 - T_2)R = (17.658 - 13.734) \times 0.10 = 0.3924 \text{ N m}$. $I\alpha = 0.010 \times (3.924 \div 0.10) = 0.3924 \text{ N m}$. **They agree.**

Problem 16 Falling rod

1. $\tau = mg(L/2) = 1.2 \times 9.81 \times 0.40 = \mathbf{4.7 \text{ N m}}$.
2. $I = (1/3)mL^2 = (1/3) \times 1.2 \times (0.80)^2 = 0.256 \text{ kg m}^2$. $\alpha = 4.709 \div 0.256 = \mathbf{18 \text{ rad/s}^2}$.
3. $a = \alpha L = 18.4 \times 0.80 = \mathbf{15 \text{ m/s}^2}$, which is 1.5 times g . The free end accelerates faster than free fall because the rod's other parts pull it round.

Problem 17 Unwinding from a cylinder

1. $Mg - T = Ma$.
2. Torque of the tension about the centre: $TR = I\alpha = \frac{1}{2}MR^2(a/R)$, so $T = \frac{1}{2}Ma$.
3. $Mg - \frac{1}{2}Ma = Ma$, so $a = (2/3)g = \mathbf{6.5 \text{ m/s}^2}$ (6.54 m/s²). $T = \frac{1}{2} \times 0.50 \times 6.54 = \mathbf{1.6 \text{ N}}$.

Problem 18 Analysing an experiment

1. Gradient = $(9.9 - 2.1) \div (0.5 - 0.1) = 7.8 \div 0.4 = \mathbf{19.5 \text{ rad/s}^2 \text{ per N m}}$.
2. The gradient is $1/I$, so $I = 1 \div 19.5 = \mathbf{0.051 \text{ kg m}^2}$.
3. $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 0.80 \times (0.36)^2 = 0.0518 \text{ kg m}^2$. Difference = $(0.0518 - 0.0513) \div 0.0518 = \mathbf{1 \%}$.

Problem 19 Spot the error

1. More mass at the rim means more rotational inertia, and for the same torque a larger I gives a smaller angular acceleration, not a larger one.
2. Hoop: $I = 4.0 \times 0.25 = 1.0 \text{ kg m}^2$, so $\alpha = 2.0 \div 1.0 = \mathbf{2.0 \text{ rad/s}^2}$. Disk: $I = 0.50 \text{ kg m}^2$, so $\alpha = 2.0 \div 0.50 = \mathbf{4.0 \text{ rad/s}^2}$.
3. $\alpha_{\text{disk}} : \alpha_{\text{hoop}} = \mathbf{2 : 1}$.

Problem 20 Child on a roundabout

1. $\tau = RF = 2.0 \times 80 = \mathbf{160 \text{ N m}}$.
2. $I = 200 + 30 \times (1.5)^2 = 267.5 \text{ kg m}^2$. $\alpha = 160 \div 267.5 = \mathbf{0.60 \text{ rad/s}^2}$.
3. $I = 200 + 30 \times (0.50)^2 = 207.5 \text{ kg m}^2$, so $\alpha = 160 \div 207.5 = \mathbf{0.77 \text{ rad/s}^2}$. Moving inwards reduces the child's contribution mr^2 to the total, so the same torque gives a larger acceleration.