

Springs, pendulums and simple harmonic motion

AP Physics 1 (algebra-based) · Lesson

Quick answer: Simple harmonic motion (SHM) is repeating motion in which the net force is proportional to the displacement from equilibrium and points back toward it, $F = -kx$. A mass on a spring has period $T = 2\pi\sqrt{m/k}$ and a small-angle pendulum has $T = 2\pi\sqrt{L/g}$; neither period depends on amplitude.

Level: AP Physics 1 (algebra-based) · **Curriculum:** AP P1 Unit 7

What is simple harmonic motion?

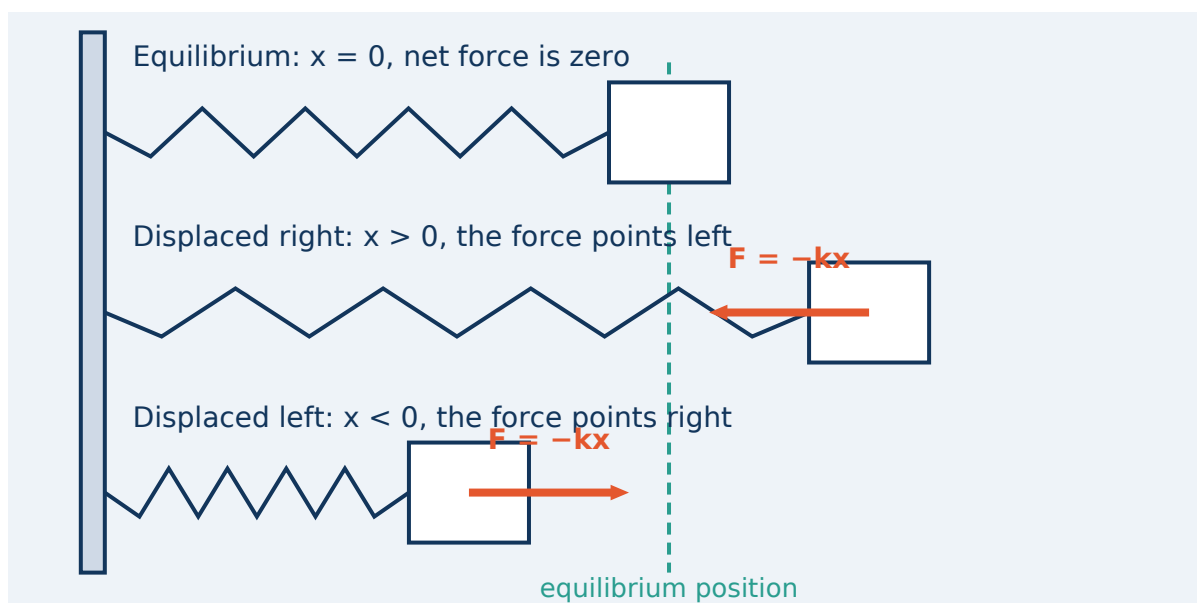
Many things move back and forth again and again: a guitar string, a child on a swing, a mass bouncing on a spring. The motion is **periodic** because it repeats exactly after a fixed time. It is **simple harmonic motion** (SHM) when the force that drives it has one special form.

Definition

An object moves with SHM when the net force on it is proportional to its displacement from the equilibrium position and is directed back toward that position. The **amplitude** A is the greatest displacement, the **period** T is the time for one full cycle, and the **frequency** $f = 1/T$ is the number of cycles per second (hertz).

What does the restoring force look like?

For an ideal spring the force is $F = -kx$. The minus sign says the force is opposite to the displacement, so it always pulls or pushes the object back to $x = 0$. Newton's second law then gives $a = -(k/m)x$, an acceleration that grows with distance from the centre.



The spring force always points back toward the equilibrium position, so its sign is opposite to the displacement.

Key law

A restoring force proportional to displacement, $F = -kx$, produces SHM. The acceleration is greatest at the turning points and zero at the centre.

How do you find the period of a spring oscillator?

A heavier mass accelerates less for the same force, so it oscillates more slowly. A stiffer spring pushes harder, so it oscillates faster. The result is independent of the amplitude.

Formula

$$T = 2\pi\sqrt{m/k} \quad \text{and} \quad f = 1/T$$

T = period (s), m = oscillating mass (kg), k = spring constant (N/m), f = frequency (Hz).

Worked example

Wanjiru hangs a 0.250 kg mass on a spring with $k = 40$ N/m and sets it bouncing vertically.

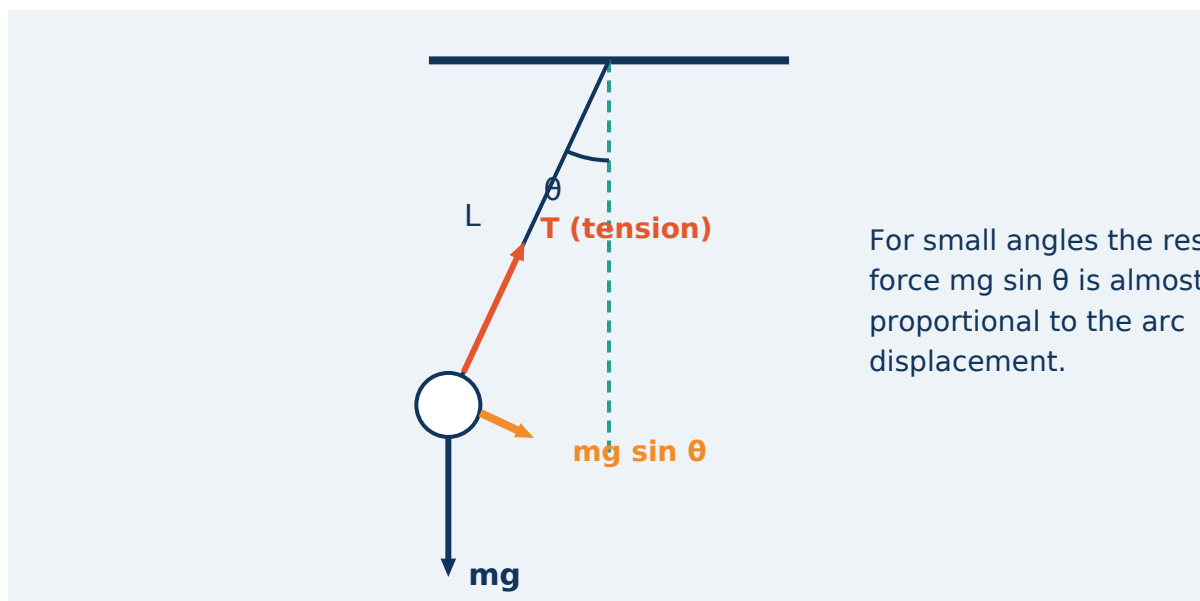
Period: $T = 2\pi\sqrt{(0.250 \div 40)} = 2\pi \times 0.0791 = 0.497$ s. Frequency: $f = 1 \div 0.497 = \mathbf{2.01}$

Hz. Doubling the amplitude would leave both numbers unchanged.

A vertical spring oscillates with the same formula. Gravity only moves the equilibrium position down by mg/k ; the restoring force measured from that new centre is still $-kx$.

How do you find the period of a pendulum?

A pendulum bob moves along an arc. For angles below about 10° the restoring force $mg \sin \theta$ is nearly proportional to the displacement, so the motion is close to SHM. The mass cancels, because the restoring force and the inertia are both proportional to m .



Only the component of the weight along the arc, $mg \sin \theta$, acts as the restoring force. The tension does no work.

Formula

$$T = 2\pi\sqrt{L/g}$$

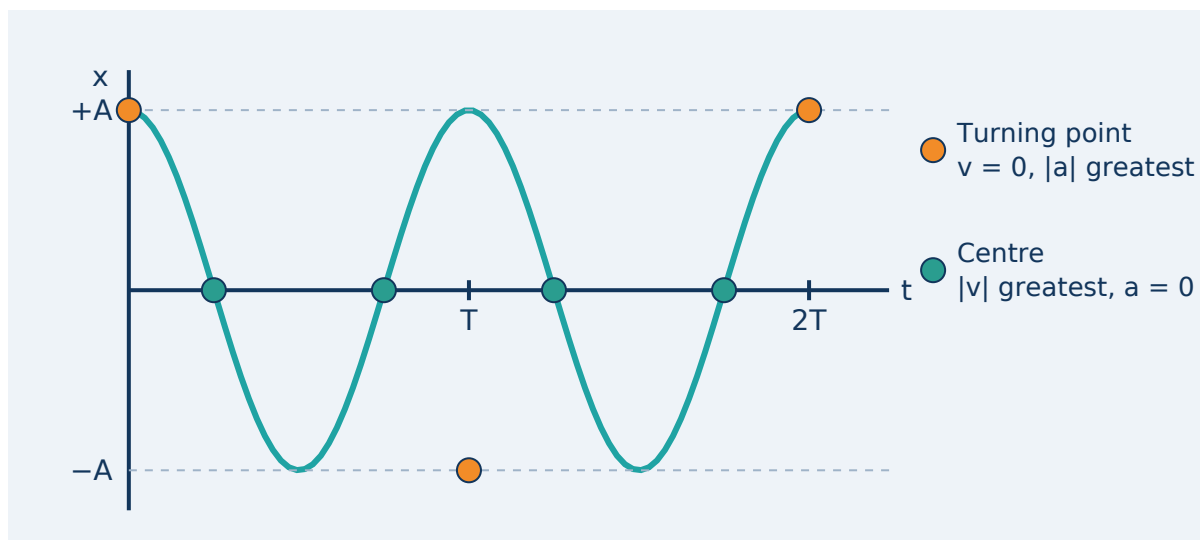
T = period (s), L = length from pivot to centre of the bob (m), g = gravitational field strength (9.81 N/kg).

Worked example

A brass lamp hangs from a 2.50 m chain in a temple in Chennai and swings through a small angle. $T = 2\pi\sqrt{(2.50 \div 9.81)} = 2\pi \times 0.5048 = \mathbf{3.17 \text{ s}}$. To halve the period the chain must be cut to a quarter of its length, because T is proportional to $\sqrt{L}$.

How do position, speed and acceleration change?

The graph below shows displacement against time. Where the graph is steepest the object is moving fastest; where it flattens at a peak the object stops for an instant and turns around.

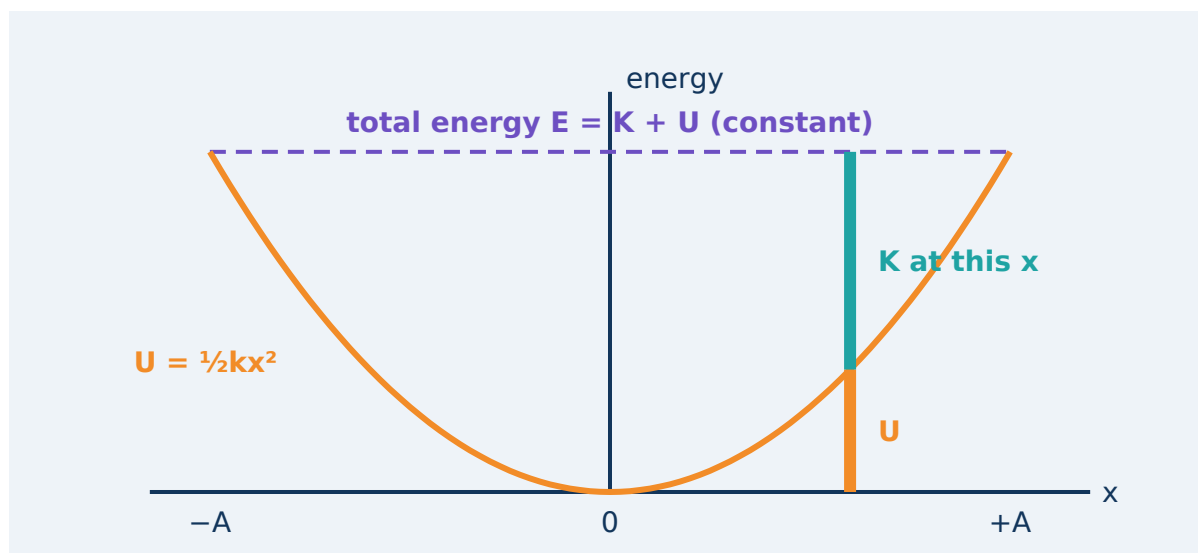


Displacement against time. The gradient of this graph is the velocity and the gradient of the velocity graph is the acceleration.

Position	Speed	Acceleration	Kinetic energy	Spring energy
$x = \pm A$ (turning point)	zero	greatest, toward centre	zero	greatest
$x = 0$ (centre)	greatest	zero	greatest	zero
in between	between	between, toward centre	$E - U$	$\frac{1}{2}kx^2$

Where does the energy go during an oscillation?

With no friction the total mechanical energy stays constant. It swaps between spring energy $U = \frac{1}{2}kx^2$ and kinetic energy $K = \frac{1}{2}mv^2$. At the turning points $E = \frac{1}{2}kA^2$, so the maximum speed is $v = A\sqrt{k/m}$.



At the turning points all the energy is stored in the spring. At the centre it is all kinetic. In between, the gap above the curve is the kinetic energy.

Worked example

A 0.80 kg glider on an air track is attached to a spring with $k = 150 \text{ N/m}$ and released from $A = 0.20 \text{ m}$. Total energy $E = \frac{1}{2} \times 150 \times 0.20^2 = 3.0 \text{ J}$. At the centre all of it is kinetic: $v = \sqrt{2 \times 3.0 \div 0.80} = 2.7 \text{ m/s}$.

Common mistake

Students often think a larger amplitude or a heavier pendulum bob changes the period of a pendulum. For small angles the period depends only on L and g . A spring oscillator depends on m and k , never on A .

Lina's tip

Sketch the x - t graph before you calculate. Gradient = velocity, curvature = acceleration: peaks mean $v = 0$, crossings of the axis mean maximum speed. Then use the formulas

only for numbers.

Key points to remember

- Restoring force: $F = -kx$, giving $a = -(k/m)x$.
- Spring oscillator: $T = 2\pi\sqrt{m/k}$; pendulum (small angles): $T = 2\pi\sqrt{L/g}$; $f = 1/T$.
- Period does not depend on amplitude; a pendulum period does not depend on mass.
- Energy: $E = \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$; maximum speed $v = A\sqrt{k/m}$ at the centre.
- Speed is zero and acceleration greatest at $x = \pm A$; the reverse at $x = 0$.

Springs, pendulums and simple harmonic motion: frequently asked questions

What is the difference between period and frequency?

Period T is the time for one complete oscillation, measured in seconds. Frequency f is the number of oscillations per second, measured in hertz. They are reciprocals: $f = 1/T$.

Why does amplitude not change the period of a spring oscillator?

A larger amplitude means a larger restoring force, so the object accelerates more, but it also has further to travel. The two effects cancel exactly for $F = -kx$, so $T = 2\pi\sqrt{m/k}$ is the same for any amplitude.

Does the mass of a pendulum bob affect its period?

No. The restoring force $mg \sin \theta$ and the inertia m both scale with the mass, so the mass cancels. Only the length L and the gravitational field strength g matter, provided the angle is small.

Where is the speed greatest in SHM?

The speed is greatest as the object passes through the equilibrium position, where the potential energy is lowest. The speed is zero at the turning points $x = \pm A$.

Does gravity change the period of a vertical spring?

No. Gravity only shifts the equilibrium position down by mg/k . Measured from the new centre the restoring force is still $-kx$, so the period is still $2\pi\sqrt{m/k}$.