

Springs, pendulums and simple harmonic motion

AP Physics 1 (algebra-based) · Worksheet answers

Full methods are shown, with formula, substitution and result. Results are given to a sensible number of significant figures, usually matching the data. Check your own working step by step, not only the final number.

Level: AP Physics 1 (algebra-based) · **Curriculum:** AP P1 Unit 7

Problem 1 Which motions are SHM?

1. **A and C** are SHM (C only approximately, because the angle is small). In B the force changes abruptly at the floor and is constant in flight. In D the motion does not move back and forth along a line about an equilibrium point.
2. **The net force must be proportional to the displacement from equilibrium and directed back toward equilibrium, $F = -kx$.**

Problem 2 Metronome timing

1. $f = 72 \div 60 \text{ s} = \mathbf{1.2 \text{ Hz}}$.
2. $T = 1 \div f = 1 \div 1.2 = \mathbf{0.83 \text{ s}}$.
3. $N = 72 \times 5.0 = \mathbf{360 \text{ cycles}}$.

Problem 3 Spring force

1. $F = kx = 120 \times 0.15 = \mathbf{18 \text{ N}}$.
2. The force is **opposite to the displacement**, pulling the end back toward the unstretched position.
3. $F = 120 \times 0.050 = \mathbf{6.0 \text{ N}}$, directed toward the equilibrium position.

Problem 4 Period of a spring

1. $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{(0.60 \div 150)} = 2\pi \times 0.0632 = \mathbf{0.40 \text{ s}}$.
2. $f = 1 \div 0.3974 = \mathbf{2.5 \text{ Hz}}$.
3. The period is **unchanged at 0.40 s**, because the period does not depend on amplitude.

Problem 5 Period of a pendulum

1. $T = 2\pi\sqrt{L/g} = 2\pi\sqrt{(0.640 \div 9.81)} = 2\pi \times 0.2554 = \mathbf{1.60 \text{ s}}$.
2. **1.60 s**, because the mass cancels from the pendulum equation.

Problem 6 Reading an x-t graph

1. The amplitude is **6.0 cm**.
2. One cycle takes **0.80 s**, so $f = 1 \div 0.80 = \mathbf{1.25 \text{ Hz}}$.
3. The speed is zero at the peaks: **$t = 0$ and $t = 0.40 \text{ s}$** (and 0.80 s). The speed is greatest as the graph crosses $x = 0$: **$t = 0.20 \text{ s}$** (or 0.60 s).

Problem 7 Spring constant from stretch

1. $m = 0.350 \text{ kg}$ and $x = 0.084 \text{ m}$.
2. At equilibrium $kx = mg$, so $k = 0.350 \times 9.81 \div 0.084 = \mathbf{40.9 \text{ N/m}}$.
3. $T = 2\pi\sqrt{(0.350 \div 40.875)} = 2\pi \times 0.0925 = \mathbf{0.58 \text{ s}}$. (Check: $T = 2\pi\sqrt{(x/g)} = 2\pi\sqrt{(0.084 \div 9.81)} = 0.58 \text{ s}$.)

Problem 8 Energy of a glider

1. $E = \frac{1}{2}kA^2 = \frac{1}{2} \times 150 \times 0.20^2 = \mathbf{3.0 \text{ J}}$.
2. At $x = 0$, $\frac{1}{2}mv^2 = E$, so $v = \sqrt{(2 \times 3.0 \div 0.80)} = \mathbf{2.7 \text{ m/s}}$.
3. $U = \frac{1}{2} \times 150 \times 0.10^2 = 0.75 \text{ J}$, so $K = 3.0 - 0.75 = 2.25 \text{ J}$ and $v = \sqrt{(2 \times 2.25 \div 0.80)} = \mathbf{2.4 \text{ m/s}}$.

Problem 9 Scaling the period

$$T = 2\pi\sqrt{(m/k)}.$$

1. T is proportional to $\sqrt{m}$, so it doubles: **2.4 s**.
2. The period does not depend on amplitude: **1.2 s**.
3. T is proportional to $1/\sqrt{k}$, so it halves: **0.60 s**.
4. The mass and the spring constant are unchanged and gravity plays no part horizontally: **1.2 s**.

Problem 10 Finding g with a pendulum

1. $T = 33.0 \div 15 = \mathbf{2.20 \text{ s}}$.
2. $T^2 = 4\pi^2 L/g$, so **$g = 4\pi^2 L/T^2$** .
3. $g = 39.48 \times 1.20 \div 2.20^2 = 47.37 \div 4.84 = \mathbf{9.79 \text{ m/s}^2}$.
4. $(9.81 - 9.79) \div 9.81 \times 100 = \mathbf{0.2 \%}$.

Problem 11 Greatest acceleration

1. $F = kA = 64 \times 0.050 = \mathbf{3.2 \text{ N}}$.
2. $a = F \div m = 3.2 \div 0.25 = \mathbf{12.8 \text{ m/s}^2}$.
3. Toward the equilibrium position, that is in the **negative x direction**, because $a = -(k/m)x$.

Problem 12 A vertical spring

1. $mg = kx$, so $x = 0.40 \times 9.81 \div 32 = \mathbf{0.123 \text{ m}}$.
2. $T = 2\pi\sqrt{(0.40 \div 32)} = 2\pi \times 0.1118 = \mathbf{0.70 \text{ s}}$.
3. The net force is $kA = 32 \times 0.050 = \mathbf{1.6 \text{ N upward}}$. The spring force is $k(0.1226 + 0.050) = \mathbf{5.5 \text{ N upward}}$, which exceeds the weight of 3.9 N by 1.6 N.

Problem 13 Reading an energy graph

1. At $x = \pm 0.10 \text{ m}$ all energy is stored in the spring, so $E = \mathbf{1.0 \text{ J}}$.
2. $E = \frac{1}{2}kA^2$, so $k = 2 \times 1.0 \div 0.10^2 = \mathbf{200 \text{ N/m}}$.
3. $U = \frac{1}{2} \times 200 \times 0.060^2 = 0.36 \text{ J}$, so $K = 1.0 - 0.36 = \mathbf{0.64 \text{ J}}$.
4. $v = \sqrt{(2 \times 0.64 \div 0.50)} = \mathbf{1.6 \text{ m/s}}$.

Problem 14 Force and work from a graph

1. $k = \Delta F \div \Delta x = 18 \div 0.060 = \mathbf{300 \text{ N/m}}$.
2. The energy is the area under the line: $\frac{1}{2} \times 0.060 \times 18 = \mathbf{0.54 \text{ J}}$.
3. The shaded trapezium has parallel sides 12 N and 18 N and width 0.020 m: $\frac{1}{2} \times (12 + 18) \times 0.020 = \mathbf{0.30 \text{ J}}$. (Check: $0.54 - \frac{1}{2} \times 300 \times 0.040^2 = 0.54 - 0.24 = 0.30 \text{ J}$.)

Problem 15 Pendulum length for a target period

1. $L = gT^2/(4\pi^2) = 9.81 \times 4.00 \div 39.48 = \mathbf{0.994 \text{ m}}$.
2. $L = 9.81 \times 1.00 \div 39.48 = \mathbf{0.248 \text{ m}}$.
3. T is proportional to $\sqrt{L}$, so halving T needs one quarter of the length ($0.994 \div 4 = 0.249 \text{ m}$), not one half.

Problem 16 Translating between graphs

1. $k = 4\pi^2m/T^2 = 39.48 \times 0.20 \div 0.16 = \mathbf{49 \text{ N/m}}$.
2. $v_{\max} = 2\pi A/T = 2\pi \times 0.050 \div 0.40 = \mathbf{0.79 \text{ m/s}}$. $a_{\max} = kA/m = 49.35 \times 0.050 \div 0.20 = \mathbf{12 \text{ m/s}^2}$.
3. The x - t graph is a cosine, so the v - t graph is an inverted sine: it starts at 0, falls to -0.79 m/s at 0.10 s, returns to 0 at 0.20 s, rises to $+0.79 \text{ m/s}$ at 0.30 s and is 0 at 0.40 s. The a - t graph is an inverted cosine: it starts at -12 m/s^2 , crosses zero at 0.10 s, reaches $+12 \text{ m/s}^2$ at 0.20 s, crosses zero at 0.30 s and returns to -12 m/s^2 at 0.40 s.

Problem 17 Designing a timing experiment

1. $0.2 \div 1.5 \times 100 = \mathbf{13 \%}$.
2. 20 swings take about 30 s, so $0.2 \div 30 \times 100 = \mathbf{0.7 \%}$.
3. Independent variable: starting angle (for example 5° , 10° , 15° , 20°). Dependent variable: period T = time for 20 swings $\div 20$. Controlled: length, bob mass, same pivot and thread. Repeat each angle three times, average, and compare the periods; they should agree within

the uncertainty for small angles.

Problem 18 Argue about a vertical spring

1. $T = 2\pi\sqrt{0.50 \div 50} = 2\pi \times 0.10 = \mathbf{0.63 \text{ s}}$.
2. $x = mg/k = 0.50 \times 9.81 \div 50 = \mathbf{0.098 \text{ m}}$.
3. **Student 2 is correct.** Gravity is a constant force, so it only moves the equilibrium position down by 0.098 m, where weight and spring force balance. Measured from there, the net force is $-kx$, the same restoring law as the horizontal case, so the acceleration pattern and the period $2\pi\sqrt{m/k} = 0.63 \text{ s}$ are the same.

Problem 19 Clay dropped on a block

1. $v = A\sqrt{k/m} = 0.30 \times \sqrt{100 \div 0.90} = \mathbf{3.16 \text{ m/s}}$.
2. Horizontal momentum is conserved: $v' = 3.162 \div 2 = \mathbf{1.58 \text{ m/s}}$. Then $\omega' = \sqrt{100 \div 1.80} = 7.454 \text{ rad/s}$ and $A' = v'/\omega' = 1.581 \div 7.454 = \mathbf{0.21 \text{ m}}$ (that is $0.30 \div \sqrt{2}$).
3. $T' = 2\pi\sqrt{1.80 \div 100} = \mathbf{0.84 \text{ s}}$. Energy before: $\frac{1}{2} \times 100 \times 0.30^2 = 4.5 \text{ J}$; after: $\frac{1}{2} \times 100 \times 0.2121^2 = 2.25 \text{ J}$, so **half** the energy is lost in the inelastic collision.

Problem 20 Mass and period data

1. $T^2 = \mathbf{0.0986, 0.181, 0.263, 0.346, 0.428 \text{ s}^2}$.
2. Gradient = $(0.4277 - 0.0986) \div (0.50 - 0.10) = \mathbf{0.823 \text{ s}^2/\text{kg}}$.
3. $T^2 = (4\pi^2/k)m$, so $k = 39.48 \div 0.823 = \mathbf{48 \text{ N/m}}$.
4. The line does not pass through the origin (intercept about 0.016 s^2) because part of the spring's own mass also oscillates, so the effective oscillating mass is slightly larger than the hanging mass.