

# Simple harmonic motion and the physical pendulum

AP Physics C (Mechanics and E&M, calculus-based) · Test answers (mark scheme)

The model answers show where each mark is earned. Derivations are marked on the logic of each step, so a correct final expression without the steps does not earn full credit. Numerical answers need a unit.

**Level:** AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 7

## Question 1 Spring oscillator (2 marks)

$$\omega = \sqrt{k/m} = \sqrt{100 / 0.25} = \mathbf{20 \text{ rad/s}}. T = 2\pi/\omega = \mathbf{0.31 \text{ s}}.$$

Marking:  $\omega$  (1);  $T$  (1).

## Question 2 Calculus on $x(t)$ (3 marks)

a.  $v = -0.60 \sin 10t$ , so  $v_{\max} = \mathbf{0.60 \text{ m/s}}$ .

b.  $a = -6.0 \cos 10t$ , so  $a_{\max} = \mathbf{6.0 \text{ m/s}^2}$ .

c.  $x = 0.060 \cos(1.0) = 0.060 \times 0.540 = \mathbf{0.032 \text{ m}}$ .

Marking: one mark per part.

## Question 3 Amplitude and period (2 marks)

$\omega^2 = k/m$  depends only on the spring constant and the mass. The solution  $x = A \cos(\omega t + \phi)$  has  $A$  as a free constant set by the start conditions. **Changing  $A$  therefore leaves  $T = 2\pi/\omega$  unchanged.**

Marking:  $\omega$  depends on  $k$  and  $m$  (1);  $A$  set by initial conditions (1).

## Question 4 Energy and speed (3 marks)

a.  $E = \frac{1}{2}kA^2 = \frac{1}{2} \times 250 \times 0.0064 = \mathbf{0.80 \text{ J}}$ .

b.  $U = \frac{1}{2} \times 250 \times 0.0016 = \mathbf{0.20 \text{ J}}$ .

c.  $K = 0.60 \text{ J}$ , so  $v = \sqrt{(2 \times 0.60 / 0.40)} = \mathbf{1.7 \text{ m/s}}$ .

Marking: one mark per part.

## Question 5 Physical pendulum (3 marks)

The torque about  $O$  is  $\tau = -mgd \sin\theta$ . For small  $\theta$ ,  $\sin\theta \approx \theta$ , so  $\tau = -mgd \theta$ . Newton's second law for rotation gives  $I d^2\theta/dt^2 = -mgd \theta$ , which is  $d^2\theta/dt^2 = -(mgd/I)\theta$ . **This matches  $d^2\theta/dt^2 = -\omega^2\theta$  with  $\omega = \sqrt{(mgd/I)}$ .**

Marking: torque expression (1); small-angle step (1); comparison with SHM equation (1).

### Question 6 Swinging rod (3 marks)

$I = \frac{1}{3}ML^2$  and  $d = L/2$ , so  $T = 2\pi\sqrt{I / (Mgd)} = 2\pi\sqrt{2L / 3g} = 2\pi\sqrt{1.2 / 29.43} = 2\pi \times 0.2019 = \mathbf{1.27 \text{ s}}$ .

Marking:  $I$  and  $d$  (1); formula (1); answer with unit (1).

### Question 7 Doubling the period (2 marks)

$T \propto \sqrt{L}$ , so doubling  $T$  needs four times the length:  $\mathbf{L = 4.0 \text{ m}}$ .

Marking:  $T \propto \sqrt{L}$  (1); answer (1).

### Question 8 Verify a solution (2 marks)

$dx/dt = A\omega \cos \omega t$ , so  $d^2x/dt^2 = -A\omega^2 \sin \omega t = \mathbf{-\omega^2 x}$ .

Marking: first derivative (1); second derivative and conclusion (1).