

Simple harmonic motion and the physical pendulum

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet

Name: _____ Date: _____ Class: _____

These problems train the calculus view of oscillations: writing and solving $d^2x/dt^2 = -\omega^2x$, differentiating $x(t)$, tracking energy, and applying the same method to pendulums and rotating bodies. Contexts range from springs and clocks to a swinging door and a metre stick. Level 1 problems practise formulas and signs, level 2 problems combine initial conditions with energy, and level 3 problems need rotational dynamics, derivations and a clear argument, as in AP Physics C: Mechanics unit 7.

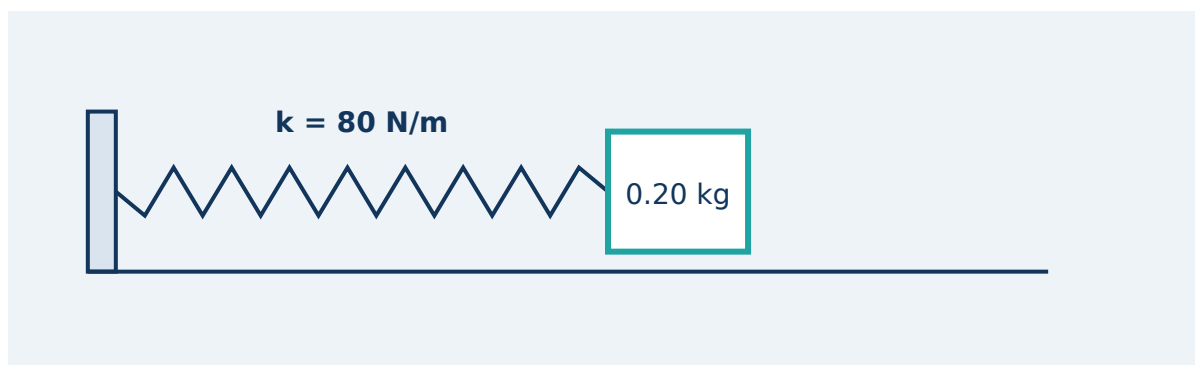
Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 7

Problem 1 Reading an equation (easy)

A particle moves with $x = 0.12 \cos(8.0t)$, with x in metres and t in seconds.

1. State the amplitude and the angular frequency.
2. Calculate the period.
3. Calculate the frequency.

Problem 2 Spring and block (easy)



Problem 2: a spring and block.

A 0.20 kg block is attached to a spring of spring constant 80 N/m on a smooth table (figure).

1. Calculate the angular frequency.
2. Calculate the period.

Problem 3 Pendulum on a cable (easy)

A heavy ball hangs from a cable 2.50 m long and swings through a small angle.

1. Calculate the period.

2. State what happens to the period if the ball's mass is doubled.

Problem 4 Energy of a spring system (easy)

A spring ($k = 300 \text{ N/m}$) is stretched 0.050 m from equilibrium and released with a block attached.

1. Calculate the total mechanical energy.
2. State the block's kinetic energy as it passes the equilibrium position.

Problem 5 Sign of the acceleration (easy)

A block oscillates on a spring.

1. State the sign of its acceleration when x is positive and at its maximum.
2. State its acceleration and its speed at $x = 0$.
3. Explain, using $a = -\omega^2 x$, why the acceleration is always directed to the equilibrium position.

Problem 6 Scaling the period (easy)

A spring-block system has period T .

1. State the new period if the mass is quadrupled.
2. State the new period if the amplitude is doubled.
3. State the new period if the spring constant is quadrupled.

Problem 7 Differentiating $x(t)$ (medium)

A particle moves with $x = 0.20 \cos(5.0t)$ (SI units).

1. Find $v(t)$ and $a(t)$.
2. Calculate x , v and a at $t = 0.30 \text{ s}$.

Problem 8 Finding A and ϕ (medium)

A block oscillates with $\omega = 10 \text{ rad/s}$. At $t = 0$ it is at $x = 0.030 \text{ m}$ moving with $v = +0.40 \text{ m/s}$.

1. Calculate the amplitude.
2. Find the phase constant ϕ in $x = A \cos(\omega t + \phi)$.

Problem 9 Speed at a given position (medium)

A 0.40 kg block on a spring with $k = 160 \text{ N/m}$ has amplitude 0.10 m .

1. Calculate ω .
2. Calculate its speed when $x = 0.060 \text{ m}$.

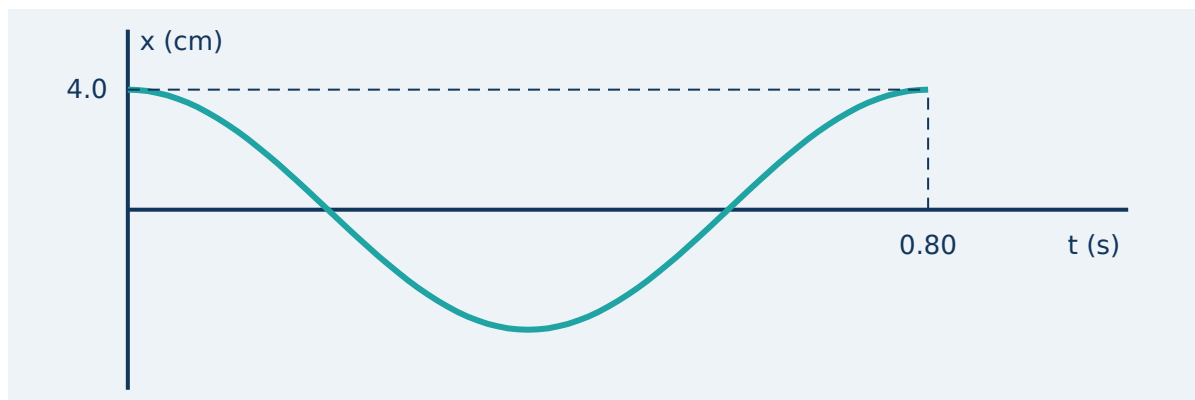
Problem 10 Vertical spring (medium)

Wanjiru in Nairobi hangs a 0.30 kg mass from a coil spring. At rest the coil lengthens by

0.060 m.

1. Calculate the stiffness of the spring.
2. She tugs the mass down and lets go. Calculate the period of the up-and-down motion.

Problem 11 Graph reading (medium)



Problem 11: displacement of a block on a spring.

The graph shows the displacement of a block on a spring.

1. Read the amplitude and the period.
2. Calculate the maximum speed.
3. Calculate the maximum acceleration.

Problem 12 Two springs (medium)

A 0.50 kg mass is attached to two springs of constants 60 N/m and 90 N/m.

1. Calculate the period if the springs act in parallel.
2. Calculate the period if they are joined in series.

Problem 13 Equal kinetic and potential energy (medium)

A block oscillates with amplitude 0.14 m. At what displacement are its kinetic and potential energies equal?

Problem 14 Spot the error (medium)

A student states: 'The period of a pendulum is $T = 2\pi\sqrt{L/g}$ at every amplitude, so a pendulum released at 60° has the same period as one released at 5° .'

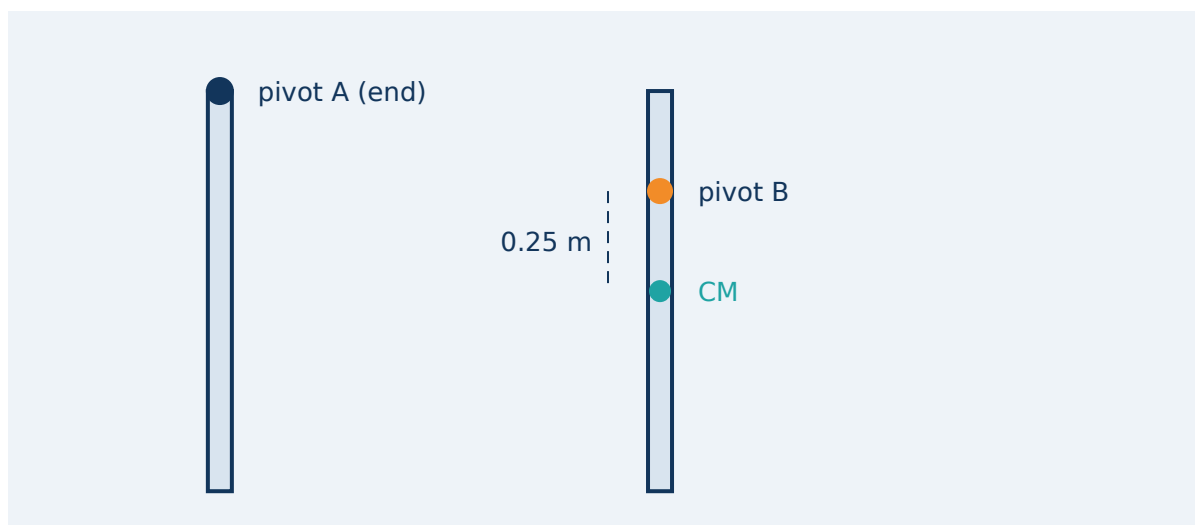
1. Identify the error.
2. State the correct comparison.

Problem 15 Pendulum on the Moon (medium)

A 0.80 m simple pendulum is taken to the Moon, where $g = 1.62 \text{ m/s}^2$.

1. Calculate its period on Earth.
2. Calculate its period on the Moon.

Problem 16 Rod as a physical pendulum (hard)

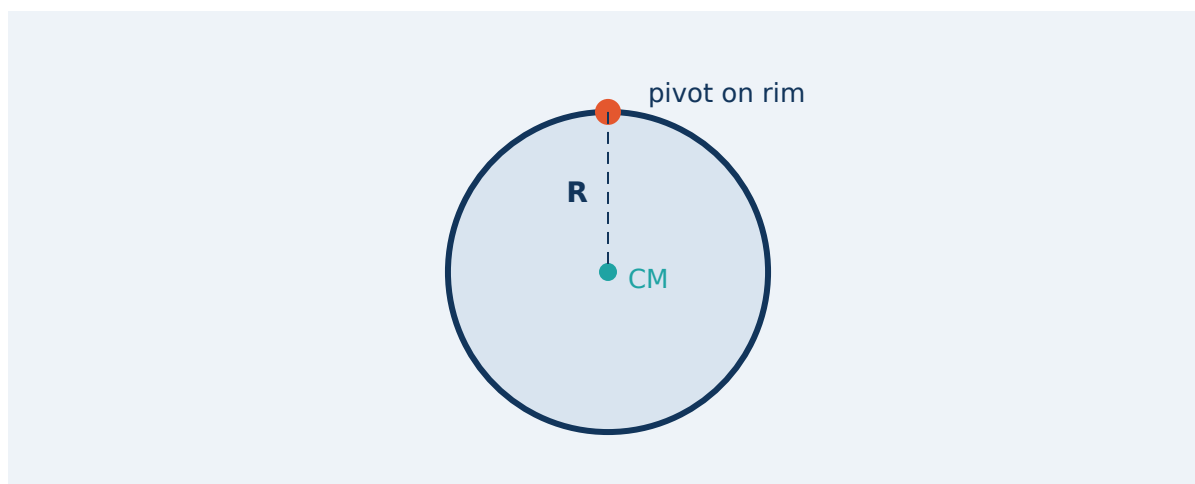


Problem 16: the same rod pivoted at its end and at 0.25 m from the centre.

A uniform rod 1.00 m long swings as a pendulum about pivot A at one end, or about pivot B, 0.25 m from the centre (figure).

1. Calculate the period about A.
2. Calculate the period about B.
3. State which is shorter and why.

Problem 17 Disc pivoted on its rim (hard)



Problem 17: a disc swinging about its rim.

A uniform disc of mass M and radius 0.20 m hangs from a horizontal pin through a point on its rim (figure).

1. Show that the moment of inertia about the pin is $\frac{3MR^2}{2}$.
2. Calculate the angular frequency and period of small oscillations.

Problem 18 Torsion pendulum (hard)

A disc with $I = 0.0040 \text{ kg m}^2$ hangs from a wire and oscillates in torsion with a period of 1.2

s. The restoring torque is $\tau = -\kappa\theta$.

1. Calculate κ .
2. The angular amplitude is 0.25 rad. Calculate the maximum angular speed.
3. Calculate the total energy.

Problem 19 Checking a solution (hard)

A 2.0 kg block moves with $x = 0.15 \sin(4.0t)$ (SI units).

1. Show that it satisfies $d^2x/dt^2 = -\omega^2x$ and state ω .
2. Find the spring constant of a spring that would produce this motion.
3. Calculate the maximum force and the total energy.

Problem 20 Motion from a potential energy (hard)

A 0.50 kg particle moves in one dimension with potential energy $U(x) = 2.0x^2$ J, with x in metres.

1. Find the force $F = -dU/dx$.
2. Show that the motion is simple harmonic and calculate its period.
3. The particle has total energy 0.080 J. Calculate the amplitude.