

Simple harmonic motion and the physical pendulum

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet answers

Each answer shows the formula, the substitution and the result with its unit in bold. When a derivation is required, check that your steps start from Newton's second law and end with $d^2q/dt^2 = -(\text{constant})q$. Round only at the end, and keep one extra digit in intermediate steps.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 7

Problem 1 Reading an equation

1. Comparing with $x = A \cos \omega t$: $A = \mathbf{0.12 \text{ m}}$ and $\omega = \mathbf{8.0 \text{ rad/s}}$.
2. $T = 2\pi/\omega = 6.283 / 8.0 = \mathbf{0.79 \text{ s}}$.
3. $f = 1/T = \mathbf{1.3 \text{ Hz}}$.

Problem 2 Spring and block

1. $\omega = \sqrt{k/m} = \sqrt{80 / 0.20} = \sqrt{400} = \mathbf{20 \text{ rad/s}}$.
2. $T = 2\pi/\omega = 6.283 / 20 = \mathbf{0.31 \text{ s}}$.

Problem 3 Pendulum on a cable

1. $T = 2\pi\sqrt{L/g} = 2\pi\sqrt{2.50 / 9.81} = 2\pi \times 0.5048 = \mathbf{3.2 \text{ s}}$.
2. The mass cancels in $\sqrt{L/g}$, so the period is **unchanged**.

Problem 4 Energy of a spring system

1. $E = \frac{1}{2}kA^2 = \frac{1}{2} \times 300 \times 0.050^2 = \mathbf{0.375 \text{ J}}$.
2. At $x = 0$ the potential energy is zero, so the kinetic energy is $\mathbf{0.375 \text{ J}}$.

Problem 5 Sign of the acceleration

1. $a = -\omega^2 x$ is negative, **pointing back towards $x = 0$** .
2. At $x = 0$ the acceleration is **zero** and the speed is at its maximum.
3. The minus sign means a and x always have opposite signs, so the acceleration always points towards equilibrium.

Problem 6 Scaling the period

- $T = 2\pi\sqrt{m/k}$.
1. $\sqrt{4} = 2$, so the period becomes $\mathbf{2T}$.

2. The period does not depend on amplitude, so it stays **T**.
3. The factor is $1/\sqrt{4}$, so the period becomes **T/2**.

Problem 7 Differentiating $x(t)$

1. $v = dx/dt = -1.0 \sin(5.0t)$ m/s and $a = dv/dt = -5.0 \cos(5.0t)$ m/s².
2. At $t = 0.30$ s the angle is 1.5 rad, $\cos 1.5 = 0.0707$, $\sin 1.5 = 0.9975$. So $x = \mathbf{0.014\ m}$, $v = -1.0\ \text{m/s}$ and $a = -0.35\ \text{m/s}^2$.

Problem 8 Finding A and ϕ

1. $A = \sqrt{(x_0)^2 + (v_0/\omega)^2} = \sqrt{(0.0009 + 0.0016)} = \mathbf{0.050\ m}$.
2. $\cos\phi = x_0/A = 0.60$ and $\sin\phi = -v_0/(A\omega) = -0.80$, so ϕ is in the fourth quadrant: $\phi = \mathbf{-0.93\ rad\ (-53^\circ)}$.

Problem 9 Speed at a given position

1. $\omega = \sqrt{(160 / 0.40)} = \mathbf{20\ rad/s}$.
2. $v = \omega\sqrt{(A^2 - x^2)} = 20 \times \sqrt{(0.0100 - 0.0036)} = 20 \times 0.080 = \mathbf{1.6\ m/s}$.

Problem 10 Vertical spring

1. At rest $k\Delta = mg$, so $k = 0.30 \times 9.81 / 0.060 = \mathbf{49\ N/m}$.
2. $T = 2\pi\sqrt{(m/k)} = 2\pi\sqrt{(0.30 / 49.05)} = \mathbf{0.49\ s}$. Gravity only shifts the equilibrium; it does not change T.

Problem 11 Graph reading

1. $A = \mathbf{4.0\ cm}$ and $T = \mathbf{0.80\ s}$.
2. $\omega = 2\pi/0.80 = 7.85\ \text{rad/s}$, so $v_{\max} = A\omega = 0.040 \times 7.85 = \mathbf{0.31\ m/s}$.
3. $a_{\max} = A\omega^2 = 0.040 \times 61.7 = \mathbf{2.5\ m/s^2}$.

Problem 12 Two springs

1. $k = 60 + 90 = 150\ \text{N/m}$, $T = 2\pi\sqrt{(0.50 / 150)} = \mathbf{0.36\ s}$.
2. $k = (60 \times 90) / 150 = 36\ \text{N/m}$, $T = 2\pi\sqrt{(0.50 / 36)} = \mathbf{0.74\ s}$.

Problem 13 Equal kinetic and potential energy

KE = PE means $\frac{1}{2}kx^2 = \frac{1}{2}\cdot\frac{1}{2}kA^2$, so $x^2 = A^2/2$ and $x = A/\sqrt{2} = 0.14 / 1.414 = \mathbf{\pm 0.099\ m}$.

Problem 14 Spot the error

1. The formula comes from $\sin\theta \approx \theta$, which holds only for small angles (below about 10°).
2. At 60° the restoring torque is weaker than the linear approximation, so the period is

about **7 % longer** than at 5° .

Problem 15 Pendulum on the Moon

1. $T = 2\pi\sqrt{0.80 / 9.81} = \mathbf{1.79 \text{ s}}$.
2. $T = 2\pi\sqrt{0.80 / 1.62} = \mathbf{4.42 \text{ s}}$, which is $\sqrt{9.81/1.62} = 2.46$ times longer.

Problem 16 Rod as a physical pendulum

1. $I = \frac{1}{3}ML^2$, $d = 0.50 \text{ m}$: $T = 2\pi\sqrt{(\frac{1}{3} / (9.81 \times 0.50))} = \mathbf{1.64 \text{ s}}$.
2. $I = (1/12)ML^2 + Md^2 = M(0.0833 + 0.0625) = 0.1458 M$; $T = 2\pi\sqrt{0.1458 / (9.81 \times 0.25)} = \mathbf{1.53 \text{ s}}$.
3. **Pivot B gives the shorter period**: the ratio $I/(Md)$ is 0.667 m at A but only 0.583 m at B, and $T = 2\pi\sqrt{(I/(Md))/g}$.

Problem 17 Disc pivoted on its rim

1. Parallel-axis theorem: $I = \frac{1}{2}MR^2 + MR^2 = \mathbf{3MR^2/2}$.
2. $\omega = \sqrt{(Mg R / (3MR^2/2))} = \sqrt{(2g / 3R)} = \sqrt{(2 \times 9.81 / 0.60)} = \mathbf{5.7 \text{ rad/s}}$, so $T = 2\pi/\omega = \mathbf{1.1 \text{ s}}$.

Problem 18 Torsion pendulum

1. $T = 2\pi\sqrt{I/\kappa}$, so $\kappa = 4\pi^2 I / T^2 = 39.48 \times 0.0040 / 1.44 = \mathbf{0.11 \text{ N m/rad}}$.
2. $\omega = 2\pi/1.2 = 5.24 \text{ rad/s}$, so $\dot{\theta}_{\max} = 0.25 \times 5.24 = \mathbf{1.3 \text{ rad/s}}$.
3. $E = \frac{1}{2}\kappa\theta_0^2 = \frac{1}{2} \times 0.1097 \times 0.0625 = \mathbf{3.4 \times 10^{-3} \text{ J}}$.

Problem 19 Checking a solution

1. $dx/dt = 0.60 \cos 4.0t$ and $d^2x/dt^2 = -2.4 \sin 4.0t = -(16)(0.15 \sin 4.0t) = -16x$, so $\omega^2 = 16$ and $\omega = \mathbf{4.0 \text{ rad/s}}$.
2. $k = m\omega^2 = 2.0 \times 16 = \mathbf{32 \text{ N/m}}$.
3. $F_{\max} = kA = 32 \times 0.15 = \mathbf{4.8 \text{ N}}$ and $E = \frac{1}{2}kA^2 = \frac{1}{2} \times 32 \times 0.0225 = \mathbf{0.36 \text{ J}}$.

Problem 20 Motion from a potential energy

1. $F = -dU/dx = \mathbf{-4.0x \text{ N}}$, so $k = 4.0 \text{ N/m}$.
2. $F = -kx$, so the motion is SHM with $\omega = \sqrt{(4.0 / 0.50)} = 2.83 \text{ rad/s}$ and $T = 2\pi/\omega = \mathbf{2.2 \text{ s}}$.
3. At the turning point $E = U = 2.0A^2$, so $A = \sqrt{(0.080 / 2.0)} = \mathbf{0.20 \text{ m}}$.