

Oscillations, springs, pendulums and resonance

Physics grades 11-12 (A Level · IB · Class 11-12) · Lesson

Quick answer: Simple harmonic motion is repeated oscillation where the acceleration is proportional to the displacement from equilibrium and points towards it, $a = -\omega^2 x$. The motion is sinusoidal with period $T = 2\pi/\omega$. A mass on a spring has $T = 2\pi\sqrt{m/k}$; a small-angle pendulum has $T = 2\pi\sqrt{L/g}$.

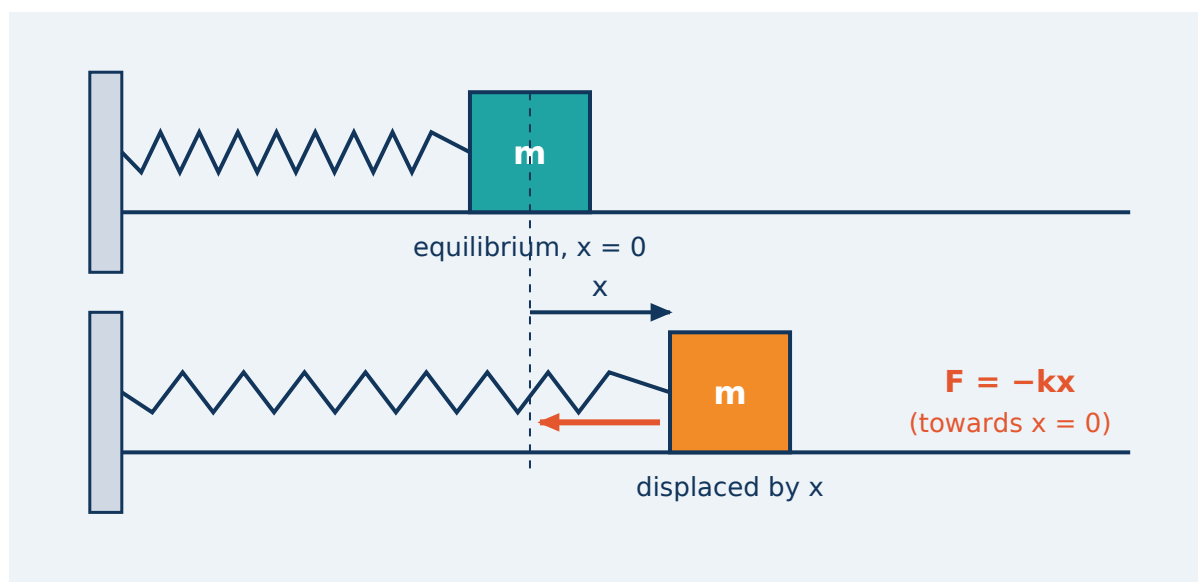
Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** IB C.1

What is simple harmonic motion?

Definition

Simple harmonic motion (SHM) is oscillation in which the acceleration is directly proportional to the displacement from a fixed equilibrium position and is always directed towards that position: $a = -\omega^2 x$.

Think of a trolley tied between two springs, a child on a swing or a phone vibrating on a table. Each one repeats the same path. What makes the motion *simple harmonic* is the minus sign and the straight-line link between acceleration and displacement. Double the displacement and the acceleration doubles; the acceleration always points back to the centre.

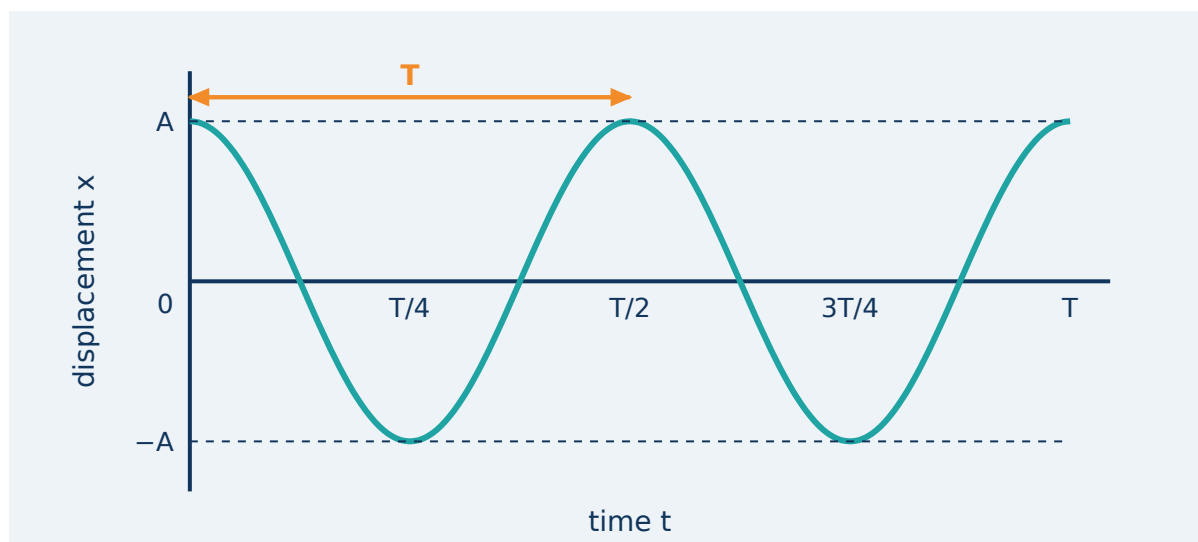


A horizontal mass-spring system. The restoring force $F = -kx$ always points towards the equilibrium position.

How do displacement, velocity and acceleration change with

time?

If timing starts at maximum displacement, the displacement follows a cosine curve. The velocity is the gradient of the displacement graph and the acceleration is the gradient of the velocity graph, so each curve is shifted a quarter of a period from the one before.



Displacement against time. The amplitude A is the maximum displacement and the period T is the time for one full cycle.

Formulas

$$x = A \cos \omega t \quad v = \pm \omega \sqrt{(A^2 - x^2)} \quad v_{\max} = \omega A \quad a_{\max} = \omega^2 A \quad \omega = 2\pi/T = 2\pi f$$

x is displacement (m), A amplitude (m), ω angular frequency (rad/s), T period (s), f frequency (Hz), v velocity (m/s), a acceleration (m/s²).

Position	Displacement	Speed	Acceleration	Kinetic energy	Potential energy
Either end	$\pm A$	0	maximum, $\omega^2 A$	0	maximum
Centre	0	maximum, ωA	0	maximum	minimum (zero)

What sets the period of a mass-spring system or a pendulum?

Newton's second law gives $ma = -kx$ for a mass on a spring, so $\omega^2 = k/m$. For a simple pendulum the restoring force along the arc is $-mg \sin \theta$, which is close to $-mg\theta$ for small angles (below about 10°), so $\omega^2 = g/L$. Neither period depends on amplitude, and a pendulum period does not depend on the mass of the bob.

Key law

Mass on a spring: $T = 2\pi\sqrt{(m/k)}$. Simple pendulum, small angles: $T = 2\pi\sqrt{(L/g)}$. In both cases the period is independent of the amplitude.

Worked example 1

A trolley of mass 0.250 kg is tied to a spring of stiffness 40.0 N/m. Calculate the period and frequency.

$$\omega = \sqrt{k/m} = \sqrt{(40.0 \div 0.250)} = \sqrt{160} = 12.65 \text{ rad/s. } T = 2\pi \div \omega = 0.497 \text{ s. } f = 1 \div T = \mathbf{2.01 \text{ Hz}} \text{ (} T = \mathbf{0.497 \text{ s}} \text{).$$

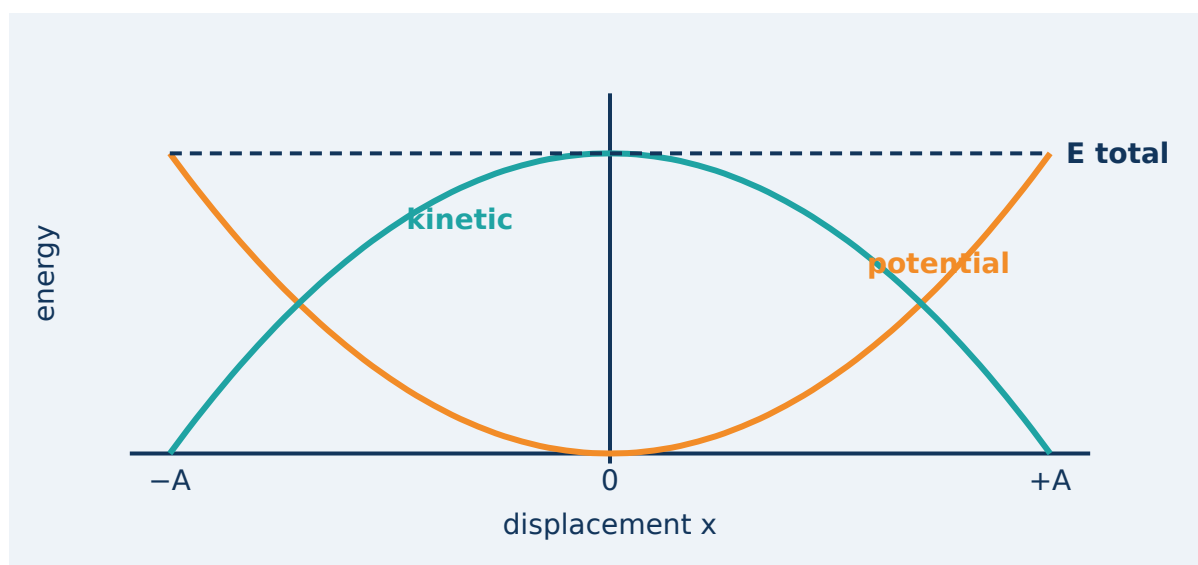
Worked example 2

A piston-like block oscillates with amplitude 0.060 m at 2.0 Hz. Find its maximum speed, maximum acceleration and speed at $x = 0.040$ m.

$$\omega = 2\pi \times 2.0 = 12.57 \text{ rad/s. } v_{\max} = \omega A = 12.57 \times 0.060 = \mathbf{0.754 \text{ m/s. } a_{\max} = \omega^2 A = 157.9 \times 0.060 = \mathbf{9.47 \text{ m/s}^2. } v = \omega \sqrt{(A^2 - x^2)} = 12.57 \times \sqrt{(0.0036 - 0.0016)} = \mathbf{0.562 \text{ m/s.}}$$

How does energy change during SHM?

With no friction the total energy stays constant while it swaps between kinetic and potential forms. The total is $E = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2A^2$. At displacement x the potential energy is $\frac{1}{2}kx^2$ and the kinetic energy is $\frac{1}{2}m\omega^2(A^2 - x^2)$. Each graph against x is a parabola, and the two curves add to a flat line.

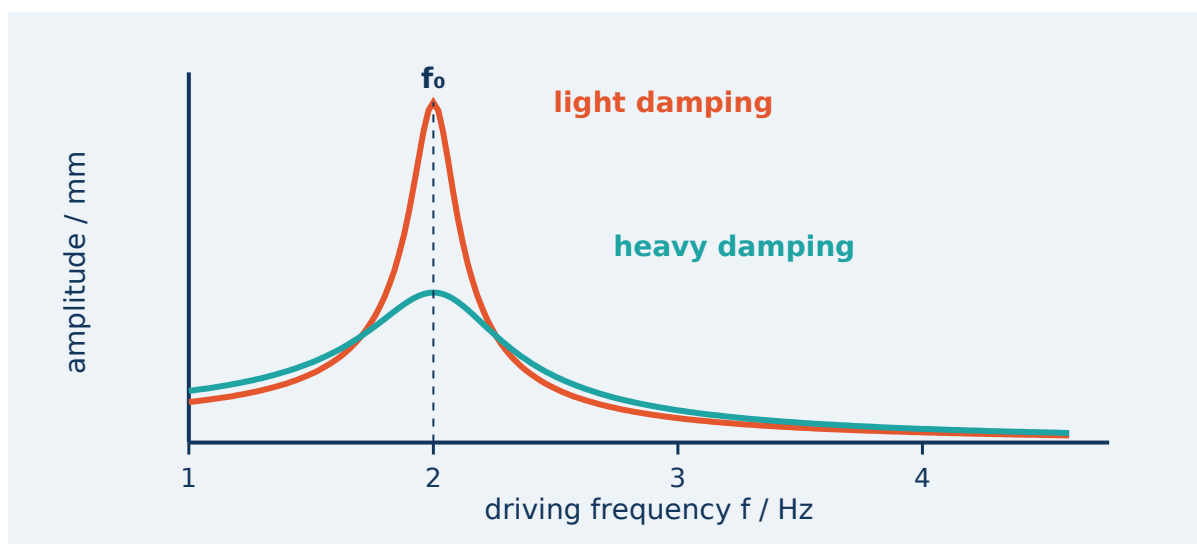


Energy against displacement. The potential energy parabola opens upwards, the kinetic energy parabola opens downwards, and their sum is constant.

What are damping and resonance?

Real oscillators lose energy to friction and air resistance, so the amplitude shrinks. This is **damping**. With light damping the oscillator keeps swinging with falling amplitude; with *critical damping* it returns to equilibrium in the shortest time without overshooting; with heavy damping it creeps back slowly. A car suspension is designed close to critical damping.

When a periodic driving force has a frequency equal to the natural frequency f_0 of the oscillator, the amplitude becomes large. This is **resonance**. Lighter damping gives a taller, sharper peak.



Resonance curves. The peak sits near the natural frequency f_0 ; more damping lowers and broadens it.

Common mistake

Students use the pendulum formula for large swings or write $T = 2\pi\sqrt{(g/L)}$. The period formula only works for small angles, and the length L goes on top.

Lina's tip

Remember the two special places: at the ends, speed is zero and acceleration is biggest; at the centre, speed is biggest and acceleration is zero.

Key points to remember

- Defining equation: $a = -\omega^2 x$, with $\omega = 2\pi/T = 2\pi f$.
- Maximum speed ωA at the centre; maximum acceleration $\omega^2 A$ at the ends.
- Mass-spring: $T = 2\pi\sqrt{(m/k)}$. Pendulum (small angles): $T = 2\pi\sqrt{(L/g)}$.
- Total energy $E = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2 A^2$, swapping between kinetic and potential.
- Resonance occurs when the driving frequency matches the natural frequency; damping lowers the peak.

Oscillations, springs, pendulums and resonance: frequently asked questions

What are the conditions for simple harmonic motion?

The acceleration must be proportional to the displacement from a fixed point and directed towards that point. Equivalent: the restoring force is proportional to displacement, $F = -kx$.

Why does the period of a pendulum not depend on its mass?

The restoring force and the inertia are both proportional to the mass, so the mass cancels in $a = -(g/L)x$. Only the length and the gravitational field strength matter.

How do you find the maximum speed in SHM?

Use $v = \omega A$, which occurs as the oscillator passes through equilibrium. You can also use $v = \omega\sqrt{(A^2 - x^2)}$ at any displacement, or energy conservation with $E = \frac{1}{2}kA^2$.

What is the difference between free and forced oscillations?

A free oscillation happens at the natural frequency after a single disturbance. A forced oscillation is driven by a periodic force and settles at the driving frequency, with large amplitude near resonance.

What is critical damping and where is it useful?

Critical damping returns the system to equilibrium in the shortest possible time without oscillating. Car suspensions, door closers and the needles of measuring instruments use damping close to critical.