

Oscillations, springs, pendulums and resonance

Physics grades 11-12 (A Level · IB · Class 11-12) · Worksheet answers

Each answer shows the formula, the substitution and the result with its unit in bold. Compare your method as well as your final number, and check that your calculator is in radians for any sine or cosine.

Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** IB C.1

Problem 1 Recognising SHM

1. Only (A): acceleration is proportional to displacement with a minus sign, so it points towards the centre. (B) points away, (C) is not proportional to x , (D) is constant.
2. $\omega^2 = 16$, so $\omega = \mathbf{4.0 \text{ rad/s}}$. $T = 2\pi/\omega = 2\pi \div 4.0 = \mathbf{1.57 \text{ s}}$.
3. Towards the equilibrium position, that is in the negative direction.

Problem 2 Swing in the park

1. $T = 36 \div 15 = \mathbf{2.4 \text{ s}}$.
2. $f = 1 \div T = 1 \div 2.4 = \mathbf{0.417 \text{ Hz}}$.
3. $\omega = 2\pi f = 2\pi \times 0.4167 = \mathbf{2.62 \text{ rad/s}}$.

Problem 3 Phone vibration motor

1. $\omega = 2\pi f = 2\pi \times 160 = \mathbf{1005 \text{ rad/s}}$ (about $1.01 \times 10^3 \text{ rad/s}$).
2. $T = 1 \div 160 = 0.00625 \text{ s} = \mathbf{6.25 \text{ ms}}$.
3. $160 \times 0.25 = \mathbf{40 \text{ oscillations}}$.

Problem 4 Pendulum in Accra

1. $T = 2\pi\sqrt{L/g} = 2\pi\sqrt{0.80 \div 9.81} = 2\pi \times 0.2856 = \mathbf{1.79 \text{ s}}$.
2. $t = 25 \times 1.794 = \mathbf{44.9 \text{ s}}$.

Problem 5 Spring balance

1. $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{0.40 \div 25} = 2\pi \times 0.1265 = \mathbf{0.795 \text{ s}}$.
2. It stays the same, because the period of SHM is independent of amplitude.
3. $f = 1 \div 0.7948 = \mathbf{1.26 \text{ Hz}}$.

Problem 6 Reading a displacement graph

1. Amplitude = **8.0 cm**.
2. Period = 1.6 s, $f = 1 \div 1.6 = \mathbf{0.625 \text{ Hz}}$.
3. Speed is greatest when the float passes $x = 0$ (steepest gradient), at **0.4 s and 1.2 s**.
4. $\omega = 2\pi \div 1.6 = 3.927 \text{ rad/s}$. $v_{\max} = \omega A = 3.927 \times 0.080 = \mathbf{0.314 \text{ m/s}}$.

Problem 7 Maximum speed and acceleration

1. $v_{\max} = \omega A = 20 \times 0.030 = \mathbf{0.60 \text{ m/s}}$.
2. $a_{\max} = \omega^2 A = 400 \times 0.030 = \mathbf{12 \text{ m/s}^2}$.
3. Maximum speed at the equilibrium position; maximum acceleration at the ends of the motion, where the speed is momentarily zero.

Problem 8 Stiffness from the period

1. $T = 2\pi\sqrt{m/k}$, so $k = 4\pi^2 m/T^2$.
2. $k = 4\pi^2 \times 0.50 \div 0.90^2 = 19.74 \div 0.81 = \mathbf{24.4 \text{ N/m}}$.
3. $T = 2\pi\sqrt{(2.0 \div 24.37)} = 2\pi \times 0.2865 = \mathbf{1.8 \text{ s}}$ (twice the original, since the mass is four times larger).

Problem 9 Measuring g with a pendulum

1. $T = 40.0 \div 20 = \mathbf{2.00 \text{ s}}$.
2. $g = 4\pi^2 L/T^2 = 4\pi^2 \times 0.995 \div 2.00^2 = 39.28 \div 4.00 = \mathbf{9.82 \text{ m/s}^2}$.
3. A reaction-time error of about 0.2 s is a much smaller fraction of 40 s than of 2 s, so the percentage uncertainty in T is reduced.

Problem 10 Speed at a given displacement

1. $\omega = 2\pi \div 0.80 = \mathbf{7.854 \text{ rad/s}}$.
2. $v = \omega\sqrt{A^2 - x^2} = 7.854 \times \sqrt{(5.0^2 - 3.0^2)} \text{ cm} = 7.854 \times 4.0 \text{ cm} = 31.4 \text{ cm/s} = \mathbf{0.314 \text{ m/s}}$.
3. $v_{\max} = \omega A = 7.854 \times 5.0 = 39.3 \text{ cm/s} = \mathbf{0.393 \text{ m/s}}$, which is larger, as expected because 3.0 cm is not the centre.

Problem 11 Energy of a spring oscillator

1. $E = \frac{1}{2}kA^2 = \frac{1}{2} \times 60 \times 0.10^2 = \mathbf{0.3 \text{ J}}$.
2. At the centre all energy is kinetic: $\frac{1}{2}mv^2 = E$, so $v = \sqrt{(2E/m)} = \sqrt{(2 \times 0.3 \div 0.30)} = \mathbf{1.41 \text{ m/s}}$.
3. $E_p = \frac{1}{2} \times 60 \times 0.060^2 = 0.108 \text{ J}$, so $E_k = 0.3 - 0.108 = \mathbf{0.192 \text{ J}}$.

Problem 12 Reading an energy graph

1. Total energy = **0.40 J**; amplitude = **0.20 m**.
2. $E = \frac{1}{2}kA^2$, so $k = 2E/A^2 = 2 \times 0.40 \div 0.20^2 = 0.80 \div 0.040 = \mathbf{20 \text{ N/m}}$.

3. $E_p = \frac{1}{2} \times 20 \times 0.10^2 = 0.10 \text{ J}$, so $E_k = 0.40 - 0.10 = \mathbf{0.30 \text{ J}}$.

Problem 13 Hanging mass

1. At equilibrium $kx = mg$, so $k = mg/e = 0.20 \times 9.81 \div 0.049 = \mathbf{40 \text{ N/m}}$.
2. $T = 2\pi\sqrt{(m/k)} = 2\pi\sqrt{(0.20 \div 40.04)} = \mathbf{0.444 \text{ s}}$.
3. Since $k = mg/e$, $m/k = e/g$, so $T = 2\pi\sqrt{(e/g)} = 2\pi\sqrt{(0.049 \div 9.81)} = 0.444 \text{ s}$, the same value.

Problem 14 Displacement at a given time

1. $\omega = 2\pi \times 2.5 = \mathbf{15.71 \text{ rad/s}}$ (5π).
2. $\omega t = 15.71 \times 0.050 = 0.7854 \text{ rad}$ ($\pi/4$), so $x = 4.0 \cos(0.7854) = \mathbf{2.83 \text{ mm}}$.
3. $x = 0$ when $\omega t = \pi/2$, so $t = (\pi/2) \div 15.71 = \mathbf{0.10 \text{ s}}$ (a quarter of the 0.40 s period).

Problem 15 Clock on the Moon

1. $T \propto 1/\sqrt{g}$, so $T_{\text{Moon}} = 1.00 \times \sqrt{(9.81 \div 1.62)} = \mathbf{2.46 \text{ s}}$.
2. The mass-spring period is unchanged: $T = 2\pi\sqrt{(m/k)}$ contains no g .
3. A pendulum is driven by the component of weight, which falls on the Moon. A spring force depends on extension only, so the restoring force at a given displacement stays the same.

Problem 16 Two springs combined

1. Parallel: each spring stretches by the same amount, so the forces add and $k = 30 + 30 = \mathbf{60 \text{ N/m}}$.
2. Series: each spring carries the full force and the extensions add, so $1/k = 1/30 + 1/30$ and $k = \mathbf{15 \text{ N/m}}$.
3. Parallel: $T = 2\pi\sqrt{(0.60 \div 60)} = 2\pi \times 0.100 = \mathbf{0.628 \text{ s}}$. Series: $T = 2\pi\sqrt{(0.60 \div 15)} = 2\pi \times 0.200 = \mathbf{1.26 \text{ s}}$.

Problem 17 Phone on a shaking shelf

1. The shelf can only push the phone upwards. With no push, the phone accelerates downwards at g (its weight alone). If the shelf accelerates downwards faster than g , the phone cannot follow it and loses contact.
2. $\omega = 2\pi \times 2.0 = 12.57 \text{ rad/s}$. Leaves when $\omega^2 A = g$, so $A = g/\omega^2 = 9.81 \div 157.9 = 0.0155 \text{ m} = \mathbf{1.6 \text{ cm}}$.
3. At the **top of the motion**, where the downward acceleration is greatest.

Problem 18 Driven oscillator data

1. The peaks are at $f_0 = \mathbf{2.0 \text{ Hz}}$, so $T = 1 \div 2.0 = \mathbf{0.50 \text{ s}}$.
2. $\omega_0 = 2\pi \times 2.0 = 12.57 \text{ rad/s}$. $k = m\omega_0^2 = 0.25 \times 157.9 = \mathbf{39.5 \text{ N/m}}$.
3. Heavier damping lowers the peak and makes it broader, but it stays near the same

natural frequency.

4. Far above f_0 the driving force is out of step with the motion, so the amplitude is **much smaller** than at resonance (a small fraction of the peak value).

Problem 19 Decaying swings

1. $A = 8.0 \times 0.95^{10} = 8.0 \times 0.5987 = \mathbf{4.79 \text{ cm}}$.

2. $E/E_0 = (0.95^{10})^2 = 0.3585$, so energy lost = $1 - 0.3585 = \mathbf{64.2 \%}$.

3. $0.95^{2n} = 0.5$, so $n = \ln 0.5 \div (2 \ln 0.95) = (-0.6931) \div (-0.10259) = \mathbf{6.76 \text{ cycles}}$.

Problem 20 Putty on the block

1. $\omega = \sqrt{(200 \div 2.0)} = 10 \text{ rad/s}$. $v = \omega A = 10 \times 0.15 = \mathbf{1.5 \text{ m/s}}$.

2. The putty lands vertically, so horizontal momentum is conserved: $2.0 \times 1.5 = 3.0 \times v$, so $v = \mathbf{1 \text{ m/s}}$.

3. New $\omega = \sqrt{(200 \div 3.0)} = 8.165 \text{ rad/s}$. Passing the centre, $v = \omega A$, so $A = 1 \div 8.165 = \mathbf{0.122 \text{ m}}$. New period $T = 2\pi \div 8.165 = \mathbf{0.77 \text{ s}}$.