

# Standing waves on strings and in pipes

Physics grades 11-12 (A Level · IB · Class 11-12) · Lesson

**Quick answer:** A stationary wave forms when two identical waves travel in opposite directions and superpose, giving fixed nodes and antinodes. A string fixed at both ends has harmonics  $f = nv/2L$ . An open pipe has all harmonics  $f = nv/2L$ ; a pipe closed at one end has only odd harmonics  $f = (2n - 1)v/4L$ .

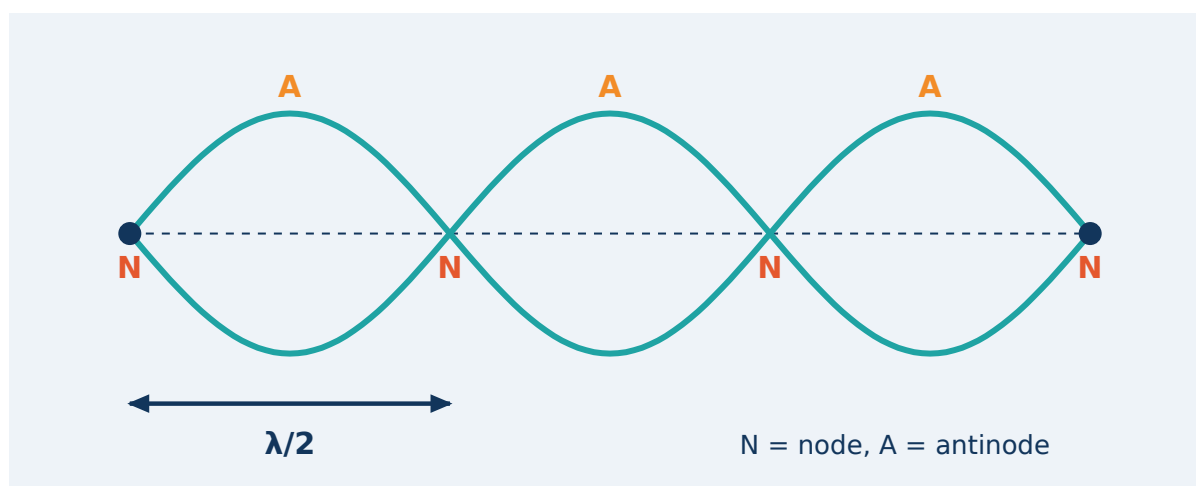
**Level:** Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** IB C.4

## What is a stationary wave?

### Definition

A **stationary (standing) wave** is formed when two progressive waves of the same frequency and amplitude travel in opposite directions and superpose. It has fixed **nodes** of zero amplitude and **antinodes** of maximum amplitude, and it does not transfer energy along the medium.

Pluck a guitar string and the wave reflects from the fixed ends. Reflected waves overlap with the incoming ones. Where they always arrive in antiphase the string never moves (a node). Where they always arrive in step the string moves with the greatest amplitude (an antinode). Between two neighbouring nodes all points move in phase; points in neighbouring loops move in antiphase.



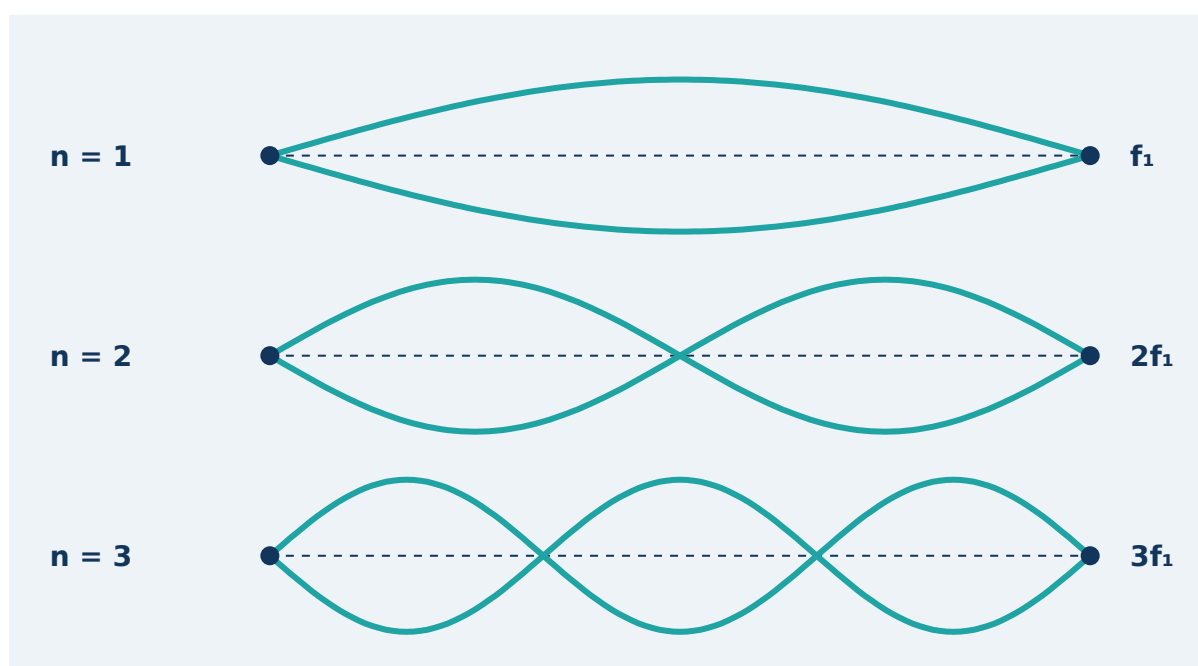
Stationary wave with three loops. Neighbouring nodes (or antinodes) are  $\lambda/2$  apart.

| Property  | Progressive wave       | Stationary wave                                  |
|-----------|------------------------|--------------------------------------------------|
| Energy    | Carried along the wave | Not transferred along; stored in the oscillation |
| Amplitude | Same for all points    | Zero at nodes, maximum at antinodes              |

| Property   | Progressive wave                 | Stationary wave                                    |
|------------|----------------------------------|----------------------------------------------------|
| Phase      | Changes steadily along the wave  | Same within a loop, opposite in neighbouring loops |
| Wavelength | Distance between points in phase | Twice the distance between adjacent nodes          |

## How do harmonics form on a stretched string?

The ends of a string are fixed, so both must be nodes. This means a whole number of half wavelengths must fit the length:  $L = n\lambda/2$ . The wave speed on the string is  $v = \sqrt{T/\mu}$ , where  $T$  is the tension and  $\mu$  the mass per unit length. Combining  $v = f\lambda$  gives the family of frequencies  $f_n = nv/2L = nf_1$ , called harmonics.



First three harmonics on a string. Each harmonic adds one more loop.

### String formulas

$$f_1 = (1/2L)\sqrt{T/\mu}, f_n = nf_1$$

$f_1$  is the fundamental frequency (Hz),  $L$  the vibrating length (m),  $T$  the tension (N),  $\mu$  the mass per unit length (kg/m),  $n = 1, 2, 3, \dots$

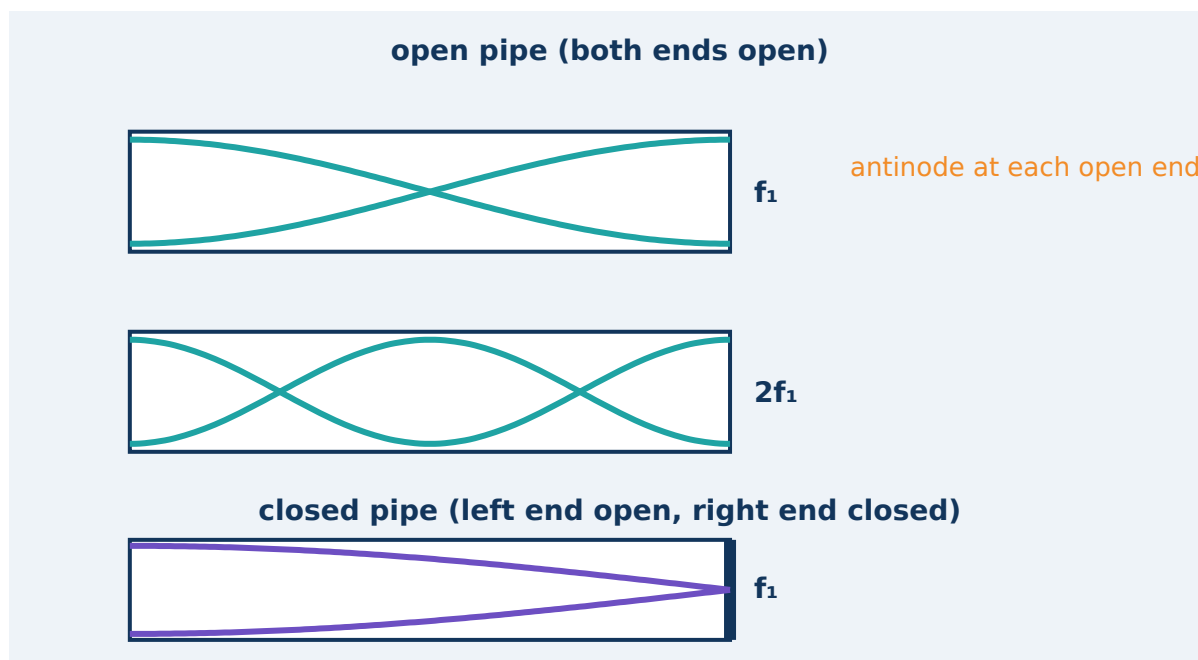
### Worked example 1

A cello string 0.69 m long has tension 120 N and mass per unit length 0.0075 kg/m. Calculate its fundamental frequency.

$$v = \sqrt{T/\mu} = \sqrt{120 \div 0.0075} = \sqrt{16\,000} = 126.5 \text{ m/s. } f_1 = v/2L = 126.5 \div (2 \times 0.69) = \mathbf{91.7 \text{ Hz.}}$$

## What are the harmonics in pipes?

In an air column the sound wave reflects at both ends. At a closed end the air cannot move: a node. At an open end the air moves freely: an antinode. An **open pipe** has an antinode at each end, so  $L = n\lambda/2$  and all harmonics occur. A **closed pipe** (closed at one end) has a node at one end and an antinode at the other, so  $L = (2n - 1)\lambda/4$  and only odd harmonics occur.



Open pipe (top two) and closed pipe (bottom). The closed pipe has a node at its closed end.

### Pipe formulas

Open pipe:  $f_n = nv/2L$ . Closed pipe:  $f_n = (2n - 1)v/4L$  (frequencies  $f_1, 3f_1, 5f_1, \dots$ ).

$v$  is the speed of sound (about 343 m/s in air at 20 °C),  $L$  the length of the pipe (m).

### Worked example 2

Compare the fundamentals of an open and a closed pipe, each 0.85 m long ( $v = 343$  m/s).

Open:  $f_1 = v/2L = 343 \div 1.70 = \mathbf{201.8 \text{ Hz}}$ . Closed:  $f_1 = v/4L = 343 \div 3.40 = \mathbf{100.9 \text{ Hz}}$ , an octave lower. The next closed-pipe resonance is  $3f_1 = \mathbf{302.6 \text{ Hz}}$ .

## Why does the real pipe seem a little longer?

The antinode at an open end lies slightly outside the pipe, by about  $0.6r$  for a pipe of radius  $r$ . This **end correction** means the effective length is  $L + 0.6r$  for a closed pipe (and  $L + 1.2r$  for an open one). Ignoring it gives frequencies that are slightly too high.

### Common mistake

Students think nodes are where the wave is "stopped" and so no energy exists there. In fact the nodes do not move, but the energy of the oscillation is stored in the loops between them. Also, never use  $f = nv/2L$  for a pipe closed at one end.

**Lina's tip**

Draw the pattern first: put a node (N) at a fixed or closed end and an antinode (A) at an open end. Then count quarter wavelengths to get the length equation.

**Key points to remember**

- Adjacent nodes (or antinodes) are  $\lambda/2$  apart.
- String fixed at both ends:  $f_n = nv/2L$ , with  $v = \sqrt{T/\mu}$ .
- Open pipe:  $f_n = nv/2L$ , all harmonics.
- Closed pipe:  $f_n = (2n - 1)v/4L$ , odd harmonics only.
- End correction: effective length  $L + 0.6r$  for a closed pipe of radius  $r$ .

## Standing waves on strings and in pipes: frequently asked questions

**What is the difference between a node and an antinode?**

A node is a point of zero amplitude where the two waves always cancel. An antinode is a point of maximum amplitude where the waves always reinforce. They alternate,  $\lambda/4$  apart.

**Why can a stationary wave not carry energy along the string?**

The two travelling waves carry equal energy in opposite directions, so the net flow is zero. Energy is only exchanged locally between kinetic and elastic forms within each loop.

**Why does a closed pipe have only odd harmonics?**

It needs a node at the closed end and an antinode at the open end. That fits an odd number of quarter wavelengths, so only  $f_1$ ,  $3f_1$ ,  $5f_1$  and so on are possible.

**How does tension change the pitch of a string?**

The wave speed  $v = \sqrt{T/\mu}$  grows with tension, so the fundamental frequency rises with the square root of the tension. Doubling the tension raises  $f$  by a factor of 1.41.

**What is the end correction?**

The antinode at an open end forms slightly beyond the pipe, so the pipe behaves as if longer by about 0.6 of its radius. Without it, calculated resonance frequencies are slightly too high.