

# Standing waves on strings and in pipes

Physics grades 11-12 (A Level · IB · Class 11-12) · Worksheet answers

Each answer gives the formula, the substitution and the result with unit in bold. Use  $v = 343$  m/s for sound in air unless the problem says otherwise, and remember that the pipe type decides which formula applies.

**Level:** Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** IB C.4

## Problem 1 Progressive or stationary?

1. A progressive wave has the same amplitude at every point and carries energy along the medium. A stationary wave has zero amplitude at nodes, maximum at antinodes, and transfers no net energy along the medium.
2. A node is a point of **zero amplitude**; an antinode is a point of **maximum amplitude**.
3. Adjacent nodes are  **$\lambda/2$**  apart.

## Problem 2 Loops on a rope

1.  $L = 3 \times \lambda/2$ , so  $\lambda = 2L/3 = 2 \times 3.0 \div 3 = \mathbf{2\text{ m}}$ .
2.  $\lambda/2 = \mathbf{1.0\text{ m}}$ .
3. **4 nodes** (2 ends + 2 in the middle).

## Problem 3 Guitar string harmonics

1.  $f_1 = v/2L = 260 \div (2 \times 0.65) = \mathbf{200\text{ Hz}}$ .
2.  $f_2 = 2f_1 = \mathbf{400\text{ Hz}}$ .
3.  $f_3 = 3f_1 = \mathbf{600\text{ Hz}}$ .

## Problem 4 Bamboo flute

1. An **antinode at each end**.
2.  $f_1 = v/2L = 343 \div 1.20 = \mathbf{286\text{ Hz}}$ .
3.  $f_2 = 572\text{ Hz}$  and  $f_3 = \mathbf{858\text{ Hz}}$  ( $2f_1$  and  $3f_1$ ).

## Problem 5 Closed organ pipe

1.  $f_1 = v/4L = 343 \div 1.00 = \mathbf{343\text{ Hz}}$ .
2.  $3f_1 = \mathbf{1029\text{ Hz}}$  and  $5f_1 = \mathbf{1715\text{ Hz}}$ .
3. A closed pipe needs a node at the closed end and an antinode at the open end, so an odd number of quarter wavelengths must fit;  $2f_1$  would put an antinode at both ends or a node at both ends.

**Problem 6 Reading a string diagram**

1. **3 antinodes**, so this is the 3rd harmonic.
2.  $L = 3\lambda/2$ , so  $\lambda = 2L/3 = 2 \times 1.20 \div 3 = \mathbf{0.80 \text{ m}}$ .
3.  $v = f\lambda = 150 \times 0.80 = \mathbf{120 \text{ m/s}}$ .

**Problem 7 Chocolate in the microwave**

1. Reflected microwaves form a stationary wave; hot spots are at the antinodes, where the amplitude is largest.
2. Antinodes are  $\lambda/2$  apart, so  $\lambda = 2 \times 6.1 = \mathbf{12.2 \text{ cm}}$ .
3.  $c = f\lambda = 2.45 \times 10^9 \times 0.122 = \mathbf{298900000.000 \text{ m/s}}$  (about  $3.0 \times 10^8 \text{ m/s}$ ).

**Problem 8 Sitar string**

1.  $2.0 \text{ g/m} = \mathbf{0.0020 \text{ kg/m}}$ .
2.  $v = \sqrt{T/\mu} = \sqrt{(80 \div 0.0020)} = \sqrt{40\,000} = \mathbf{200 \text{ m/s}}$ .
3.  $f_1 = v/2L = 200 \div 1.60 = \mathbf{125 \text{ Hz}}$ .

**Problem 9 Tuning the string**

1.  $f_1 = (1/2L)\sqrt{T/\mu}$ , and  $L$  and  $\mu$  are constant, so  $f_1$  is proportional to  $\sqrt{T}$ .
2.  $T \propto f^2$ , so  $T = 80 \times (150 \div 125)^2 = 80 \times 1.44 = \mathbf{115.2 \text{ N}}$ .
3.  $(115.2 - 80) \div 80 \times 100 = \mathbf{44 \%}$ .

**Problem 10 Melde string**

1.  $L = 4 \times \lambda/2$ , so  $\lambda = 2 \times 1.20 \div 4 = \mathbf{0.6 \text{ m}}$ .
2.  $v = f\lambda = 50 \times 0.60 = \mathbf{30 \text{ m/s}}$ .
3.  $T = mg = 0.50 \times 9.81 = \mathbf{4.905 \text{ N}}$ .  $\mu = T/v^2 = 4.905 \div 900 = \mathbf{0.00545 \text{ kg/m}}$  ( $5.4 \text{ g/m}$ ).

**Problem 11 Vocal tract**

1.  $f_1 = v/4L = 343 \div 0.68 = \mathbf{504 \text{ Hz}}$ ;  $f_2 = 3f_1 = \mathbf{1513 \text{ Hz}}$ ;  $f_3 = 5f_1 = \mathbf{2522 \text{ Hz}}$ .
2. One end (the glottis) is almost closed and acts as a node; the lips are open and form an antinode.
3. Only odd multiples of  $f_1$  appear.

**Problem 12 Which pipe?**

1. The closed pipe (bottom) has a node at its closed end.
2. Open:  $f_1 = 343 \div (2 \times 0.50) = \mathbf{343 \text{ Hz}}$ . Closed:  $f_1 = 343 \div (4 \times 0.50) = \mathbf{171.5 \text{ Hz}}$ .
3.  $343 \div 171.5 = \mathbf{2}$ , the open pipe is an octave higher.

### Problem 13 End correction

1.  $f_1 = 343 \div (4 \times 0.30) = \mathbf{285.8 \text{ Hz}}$ .
2.  $L_{\text{eff}} = 0.30 + 0.6 \times 0.020 = \mathbf{0.312 \text{ m}}$ .
3.  $f_1 = 343 \div (4 \times 0.312) = \mathbf{274.8 \text{ Hz}}$ , slightly lower.

### Problem 14 Warm air in a pipe

1.  $v = 331 + 12 = 343 \text{ m/s}$ ,  $f_1 = v/2L = 343 \div 1.00 = \mathbf{343 \text{ Hz}}$ .
2.  $v = 331 + 21 = 352 \text{ m/s}$ ,  $f_1 = \mathbf{352 \text{ Hz}}$ .
3.  $(352 - 343) \div 343 \times 100 = \mathbf{2.6 \%}$ .

### Problem 15 Sound against a wall

1.  $\lambda = v/f = 343 \div 1715 = \mathbf{0.2 \text{ m}}$ .
2.  $\lambda/2 = \mathbf{0.1 \text{ m}}$  (10 cm).
3. A node is at the wall, so the nearest antinode is  $\lambda/4 = \mathbf{0.05 \text{ m}}$  (5.0 cm) away.

### Problem 16 Consecutive resonances

1. For a string fixed at both ends the harmonics are  $f_1, 2f_1, 3f_1, \dots$  so neighbouring harmonics differ by  $f_1$ :  $300 - 240 = \mathbf{60 \text{ Hz}}$ .
2.  $240 \div 60 = \mathbf{4}$  and  $300 \div 60 = \mathbf{5}$ .
3.  $v = 2Lf_1 = 2 \times 1.50 \times 60 = \mathbf{180 \text{ m/s}}$ .

### Problem 17 Which type of pipe?

1.  $425 \div 255 = 5/3$ , which matches harmonics 3 and 5. In an open pipe the 4th harmonic (340 Hz) would lie between the 3rd and 5th, but nothing resonates between 255 Hz and 425 Hz, so even harmonics are missing: the pipe is closed at one end.
2.  $f_1 = 255 \div 3 = \mathbf{85 \text{ Hz}}$  (and  $425 \div 5 = 85 \text{ Hz}$ ).
3.  $L = v/4f_1 = 343 \div (4 \times 85) = \mathbf{1.01 \text{ m}}$ .

### Problem 18 Frequency against $1/L$

1.  $f_1 = v/2L = (v/2)(1/L)$  with constant  $v$ , which has the form  $y = mx$ .
2. Gradient =  $(400 - 160) \div (2.50 - 1.00) = \mathbf{160 \text{ Hz} \cdot \text{m}}$ .
3. gradient =  $v/2$ , so  $v = \mathbf{320 \text{ m/s}}$ .  $T = \mu v^2 = 0.0010 \times 102400.0000 = \mathbf{102.4 \text{ N}}$ .

### Problem 19 Spot the error

1. The student used the open-pipe formula for a closed pipe, and then doubled the frequency, but a closed pipe has only odd harmonics.
2.  $f_1 = v/4L = 343 \div (4 \times 0.80) = \mathbf{107 \text{ Hz}}$ .
3. The next resonance is  $3f_1 = \mathbf{322 \text{ Hz}}$ .

**Problem 20 A pipe filled with helium**

1.  $f_1 = v/4L = 343 \div 1.36 = \mathbf{252.2 \text{ Hz}}$ .

2.  $f_1 = 970 \div 1.36 = \mathbf{713.2 \text{ Hz}}$ .

3. The vocal tract length stays the same, but the speed of sound is higher in helium, so every resonant frequency rises in the ratio  $v_{\text{He}}/v_{\text{air}} = 2.83$ .