

Equations of motion for constant acceleration

Physics grades 11-12 (A Level · IB · Class 11-12) · Lesson

Quick answer: The suvat equations connect displacement, initial velocity, final velocity, acceleration and time for motion in a straight line with constant acceleration. List the three known quantities, choose the equation that leaves out the unwanted one, and give every vector a sign. In free fall near the Earth, $a = 9.81 \text{ m/s}^2$ downwards.

Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** A Level 3.4, IB A.1

What are the suvat equations?

When an object moves in a straight line with **constant acceleration**, five variables describe the motion: displacement s , initial velocity u , final velocity v , acceleration a and time t . Any three of them let you find the other two using the five equations of motion, usually called the suvat equations.

Definition

Uniform acceleration means that the velocity changes by the same amount in every second. The acceleration is the gradient of the velocity-time graph and the displacement is the area under it.

Formulae

$$v = u + at$$

$$s = \frac{1}{2}(u + v)t$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Symbols: s displacement (m), u initial velocity (m/s), v final velocity (m/s), a acceleration (m/s^2), t time (s).

$$v = u + at$$

missing: s

$$s = \frac{1}{2}(u + v)t$$

missing: a

$$s = ut + \frac{1}{2}at^2$$

missing: v

$$v^2 = u^2 + 2as$$

missing: t

$$s = vt - \frac{1}{2}at^2$$

missing: u

Each equation leaves out one variable, which tells you which one to choose.

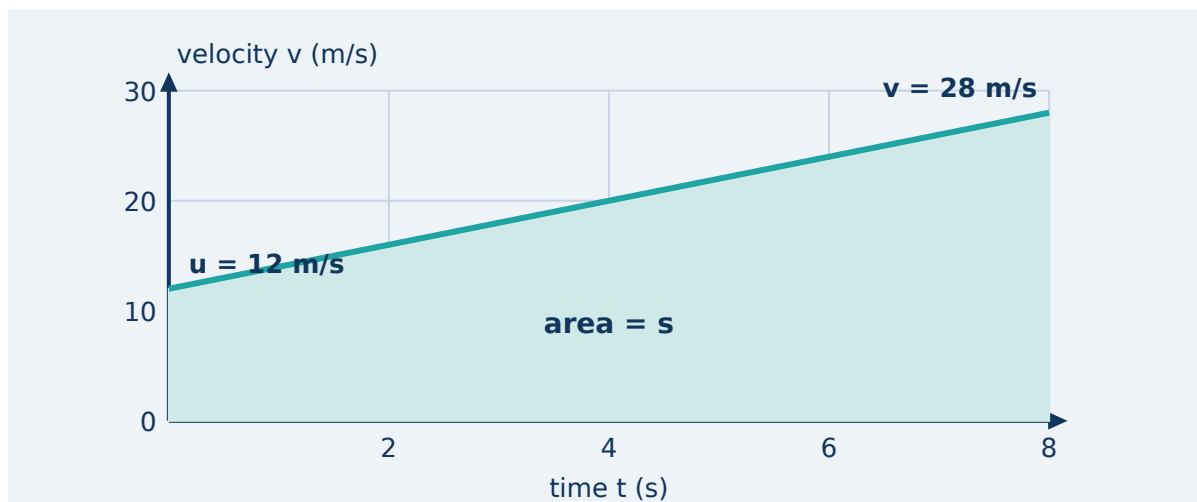
How do you choose the right equation?

Write down s , u , v , a and t , fill in the three you know, mark the one you want, and pick the equation that does not contain the fifth. Choose a positive direction first, then give every vector quantity a sign. A car braking has a negative acceleration if it moves in the positive direction.

Known	Wanted	Use
u, a, t	v	$v = u + at$
u, v, t	s	$s = \frac{1}{2}(u + v)t$
u, a, t	s	$s = ut + \frac{1}{2}at^2$
u, v, s	a	$v^2 = u^2 + 2as$

Worked example 1

A car accelerates uniformly from 12 m/s to 28 m/s in 8.0 s. Acceleration: $a = (v - u) \div t = (28 - 12) \div 8.0 = 2.0 \text{ m/s}^2$. Distance: $s = \frac{1}{2}(u + v)t = \frac{1}{2} \times 40 \times 8.0 = 160 \text{ m}$. The car has **$a = 2.0 \text{ m/s}^2$** and travels **160 m**.



The gradient of the line is the acceleration and the shaded area is the displacement.

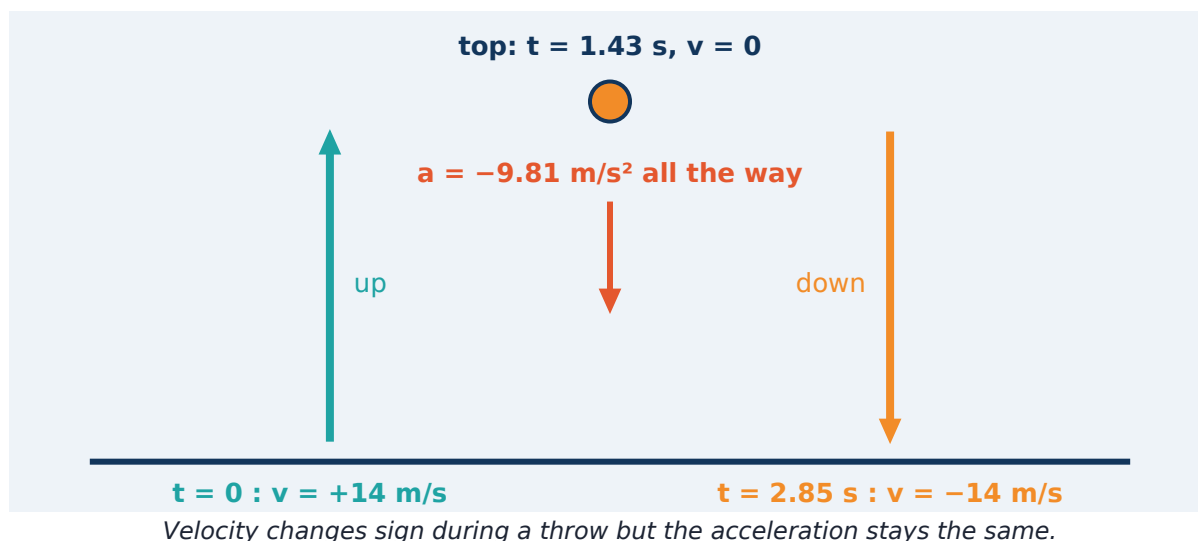
How do you treat free fall?

Near the Earth's surface, with air resistance ignored, every falling object has the same downward acceleration $g = 9.81 \text{ m/s}^2$. Treat it as a suvat problem with $a = \pm 9.81 \text{ m/s}^2$ depending on your positive direction. An object thrown upwards slows, stops for an instant at the top, then speeds up downwards, all with the same acceleration.

Worked example 2

A stone is dropped from a bridge and falls 44 m. Take down as positive: $u = 0$, $a = 9.81 \text{ m/s}^2$, $s = 44 \text{ m}$. $v^2 = 0 + 2 \times 9.81 \times 44 = 863.3$, so $v = 29.4 \text{ m/s}$. Time: $t = v \div a = 29.4 \div$

$9.81 = 3.00$ s. The stone hits at **29.4 m/s** after **3.00 s**.



Worked example 3

A ball is thrown up at 14 m/s. Take up as positive: $u = 14 \text{ m/s}$, $a = -9.81 \text{ m/s}^2$. At the top $v = 0$, so $0 = 14^2 - 2 \times 9.81 \times h$ and $h = 196 \div 19.62 = 10.0 \text{ m}$. Time to the top: $t = 14 \div 9.81 = 1.43$ s, so the flight lasts **2.85 s** and the ball rises **10.0 m**.

Common mistake

The acceleration at the top of a throw is not zero. Only the velocity is zero for an instant. Also remember that the suvat equations fail when the acceleration changes, for example when air resistance is large, or when a car changes gear.

What do the graphs tell you?

On a displacement-time graph the gradient is the velocity, and a curved line means acceleration. On a velocity-time graph the gradient is the acceleration and the area is the displacement. For a straight velocity-time line the area is a trapezium, which is exactly the equation $s = \frac{1}{2}(u + v)t$.

Lina's tip

Always draw a quick sketch with an arrow for the positive direction. Most lost marks in suvat problems come from a sign that was never decided, not from the algebra.

Key points to remember

- $v = u + at$ and $s = \frac{1}{2}(u + v)t$
- $s = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$
- Free-fall acceleration $g = 9.81 \text{ m/s}^2$ downwards
- Gradient of v - t graph = acceleration; area = displacement
- Valid only when the acceleration is constant

Equations of motion for constant acceleration: frequently asked questions

When can you use the suvat equations?

Use them only for motion in a straight line with constant acceleration. They fail if the acceleration changes, such as a falling object with growing air resistance. For a changing acceleration, use graph areas or calculus.

How do you choose which equation to use?

List s , u , v , a and t , and write which three are known and which one is wanted. Choose the equation that does not contain the remaining unused variable. If more than one works, pick the simplest.

What is the acceleration of a ball at the top of its flight?

It is still 9.81 m/s^2 downwards. Only the velocity is momentarily zero. The ball is changing its velocity at the same rate at every point, so the acceleration never switches off.

Why do I need a sign convention?

Velocity, displacement and acceleration are vectors, so direction matters. Choose a positive direction at the start, such as up, and use plus or minus signs for every quantity. This avoids contradictory answers.

How are suvat equations linked to graphs?

On a velocity-time graph the gradient is the acceleration and the area under the line is the displacement. For constant acceleration the line is straight, and its trapezium area gives $s = \frac{1}{2}(u + v)t$.