

Drag forces and terminal velocity with calculus

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet

Name: _____ Date: _____ Class: _____

These problems train the move from a velocity-dependent force to a differential equation and back to numbers. Early problems check units, directions and terminal speed; later ones ask for derivations with separation of variables, distances, and energy. They match AP Physics C: Mechanics unit 2 and first-year university mechanics.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 2

Problem 1 Drag direction (easy)

State Newton's second law for an object that feels a resistive force, then say in which direction the drag force points compared with the velocity.

1. Write the law as $m \, dv/dt = \Sigma F$.
2. Give the direction of drag.

Problem 2 Units of coefficients (easy)

A resistive force can be written $F = bv$ or $F = cv^2$.

1. Find the SI unit of b .
2. Find the SI unit of c .

Problem 3 Syrup terminal speed (easy)

Chiamaka in Lagos drops a marble of mass 0.20 kg into thick oil. The drag is $F = bv$ with $b = 0.50 \text{ kg/s}$.

1. Calculate the terminal speed.

Problem 4 Time constant (easy)

For the marble in the previous problem ($m = 0.20 \text{ kg}$, $b = 0.50 \text{ kg/s}$):

1. Calculate the time constant τ .
2. State what fraction of terminal speed the marble has reached after one time constant.

Problem 5 Paper cone (easy)

A paper cone of mass 4.0 g falls in air with $F = cv^2$, where $c = 2.0 \times 10^{-3} \text{ kg/m}$.

1. Calculate its terminal speed.

Problem 6 Zero acceleration, nonzero speed (easy)

Mei in Singapore says: "At terminal velocity the object has stopped accelerating, so it must have stopped moving."

1. Explain what is wrong with this statement.
2. State what is true about the forces.

Problem 7 Speed in oil (medium)

A bead of mass 0.10 kg is released from rest in oil with $b = 0.50 \text{ kg/s}$.

1. Calculate v_t and τ .
2. Calculate the speed 0.30 s after release.

Problem 8 Acceleration history (medium)

A sphere falls from rest through a fluid with time constant τ .

1. Write the acceleration at $t = 0$.
2. Calculate the acceleration at $t = \tau$.
3. Explain why the acceleration decreases.

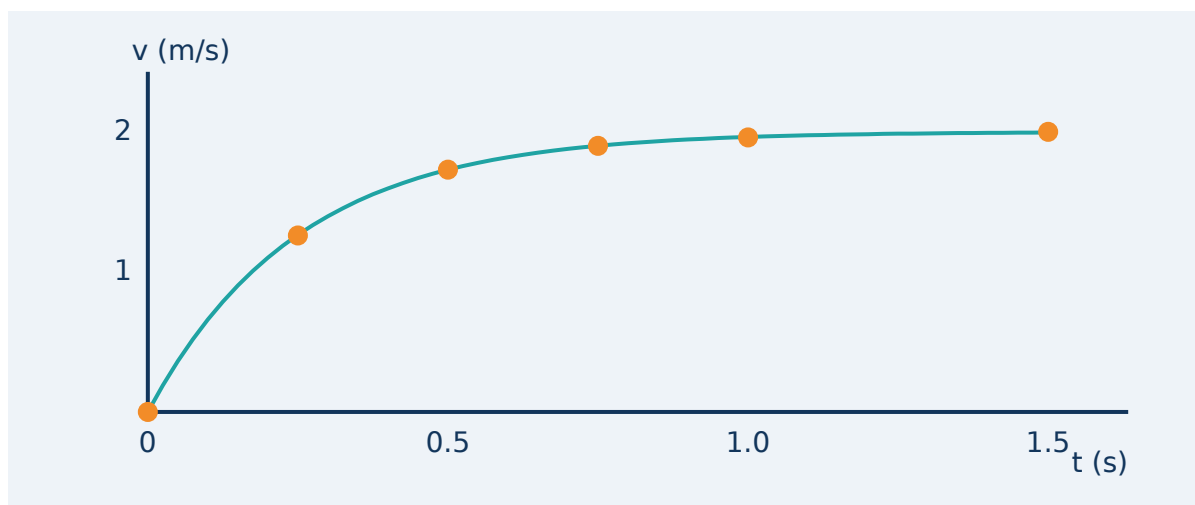
Problem 9 Time to reach 90 % (medium)

A sphere has $\tau = 0.20 \text{ s}$ in a thick liquid.

1. Derive the time at which $v = 0.90 v_t$.
2. Calculate its value.

Problem 10 Reading a graph (medium)

Aarav measures the speed of a 0.040 kg bead sinking in shampoo. Data: $(t, v) = (0, 0), (0.25 \text{ s}, 1.26 \text{ m/s}), (0.50 \text{ s}, 1.73 \text{ m/s}), (0.75 \text{ s}, 1.90 \text{ m/s}), (1.0 \text{ s}, 1.96 \text{ m/s}), (1.5 \text{ s}, 2.00 \text{ m/s})$. The points are plotted below.



Speed of a bead sinking in shampoo.

1. Estimate the terminal speed.

2. Estimate τ from the point where v is about 63 % of v_t .
3. Calculate b .

Problem 11 Parachute constant (medium)

Diego in Texas descends under a parachute at a steady 6.0 m/s. His total mass is 75 kg and $F = cv^2$.

1. Calculate c .
2. Calculate the drag force when his speed is 3.0 m/s.

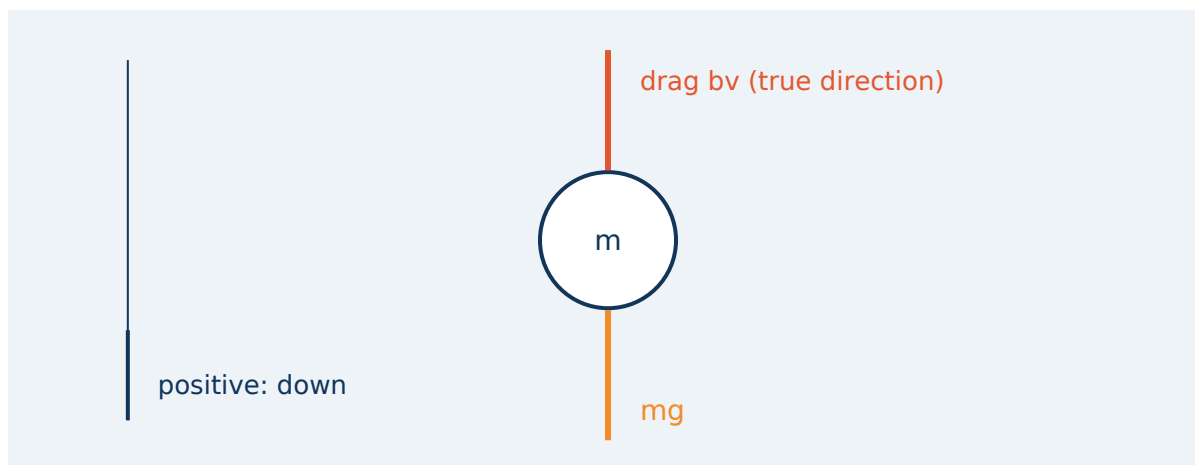
Problem 12 Doubling the mass (medium)

Two identical hollow balls are joined, doubling the mass but keeping the same shape and cross-section.

1. Predict the factor by which v_t changes with linear drag.
2. Predict the factor with quadratic drag.
3. Explain the difference.

Problem 13 Spot the sign error (medium)

A student takes down as positive and writes $m \, dv/dt = mg + bv$ for a falling ball.



Free-body diagram for the falling ball.

1. Identify the error.
2. Write the correct equation.
3. State what the wrong equation would predict for the long-term speed.

Problem 14 Verify the solution (hard)

Show that $v = v_t(1 - e^{-t/\tau})$, with $v_t = mg/b$ and $\tau = m/b$, satisfies $m \, dv/dt = mg - bv$.

1. Differentiate v .
2. Substitute into both sides.

Problem 15 Distance fallen (hard)

Wanjiru in Nairobi studies a bead with $v_t = 1.5 \text{ m/s}$ and $\tau = 0.30 \text{ s}$ starting from rest.

1. Integrate $v(t)$ to obtain $x(t)$.
2. Calculate the distance fallen in the first 0.30 s .

Problem 16 Boat coasting to rest (hard)

Bilal in Lahore switches off a 400 kg boat's engine at 6.0 m/s . Water drag is $F = bv$ with $b = 50 \text{ kg/s}$. Ignore all other forces.

1. Show that $v = v_0 e^{-bt/m}$.
2. Calculate the speed after 12 s .
3. Calculate the total distance travelled before it stops.

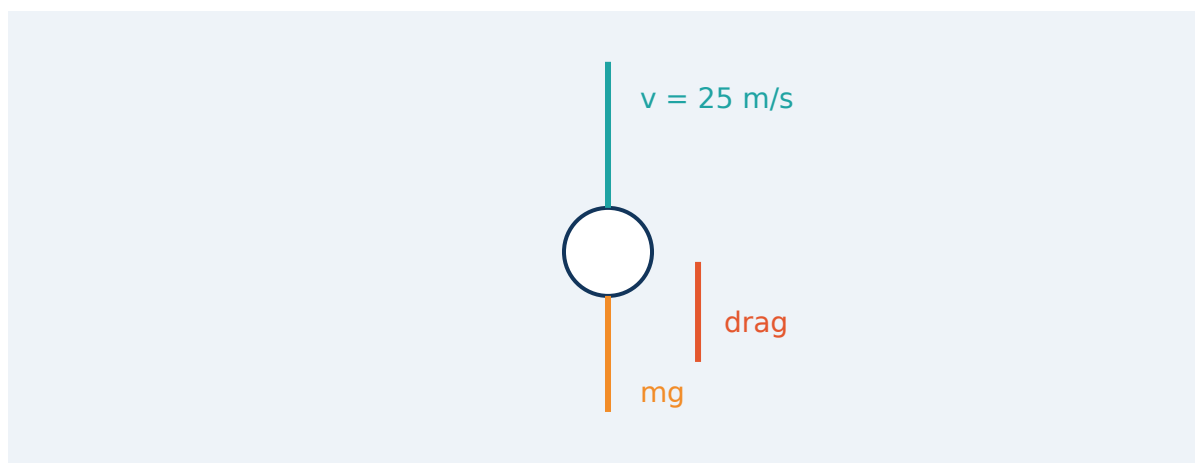
Problem 17 Skydiver with tanh (hard)

A skydiver has $v_t = 30 \text{ m/s}$ and $v = v_t \tanh(gt/v_t)$.

1. Calculate the speed after 3.0 s .
2. Calculate the speed after 3.0 s if there were no drag.
3. Comment on the difference.

Problem 18 Ball thrown upward (hard)

A ball is launched upward at 25 m/s . Drag is $F = cv^2$ with $c/m = 2.0 \times 10^{-3} \text{ m}^{-1}$.

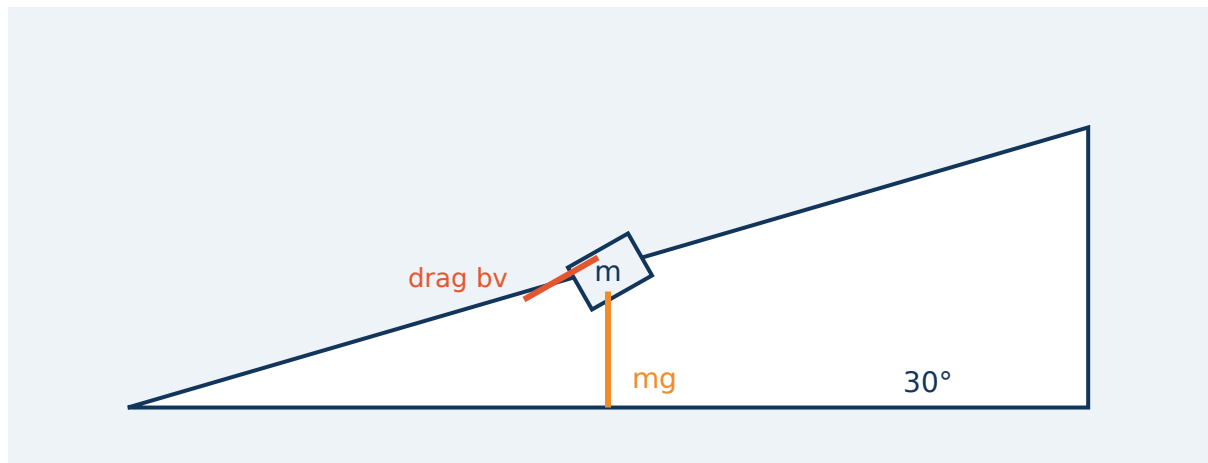


Upward launch: weight and drag act downward.

1. Calculate the deceleration at launch.
2. Calculate the terminal speed on the way down.
3. Using $v dv/dx = -g - (c/m)v^2$, show that the maximum height is $H = (m/2c) \ln(1 + cv_0^2/mg)$ and calculate it.

Problem 19 Block on an oiled ramp (hard)

A 2.0 kg block slides down a ramp inclined at 30° on a thin oil film that gives drag $F = bv$ with $b = 5.0 \text{ kg/s}$. Ignore solid friction.



Block on an oiled slope.

1. Write the equation of motion along the slope.
2. Calculate the terminal speed.
3. Calculate τ .

Problem 20 Power at terminal speed (hard)

Kofi's skydiver ($m = 80 \text{ kg}$, $v_t = 62.6 \text{ m/s}$) falls steadily.

1. Explain why the power dissipated by drag equals the rate of loss of gravitational potential energy.
2. Calculate the power dissipated by drag.