

Drag forces and terminal velocity with calculus

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet answers

Each answer shows the equation, the substitution and the result. Where a derivation is asked for, the integration steps are written out. Use $g = 9.81 \text{ m/s}^2$ and give final values to two or three significant figures, as the data allow.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 2

Problem 1 Drag direction

1. The net force equals mass times acceleration: $m \, dv/dt = \Sigma F$, and ΣF includes the drag term.
2. Drag always points **opposite to the velocity**, so it slows the object relative to the fluid.

Problem 2 Units of coefficients

1. $b = F/v$, so $\text{N} \div (\text{m/s}) = \text{kg} \cdot \text{m/s}^2 \div \text{m/s} = \text{kg/s}$.
2. $c = F/v^2$, so $\text{N} \div (\text{m}^2/\text{s}^2) = \text{kg/m}$.

Problem 3 Syrup terminal speed

At terminal speed $mg = bv_t$, so $v_t = mg/b = (0.20 \times 9.81)/0.50 = \textbf{3.9 m/s}$.

Problem 4 Time constant

1. $\tau = m/b = 0.20/0.50 = \textbf{0.40 s}$.
2. $v = v_t(1 - e^{-1}) = 0.632 v_t$, so about **63 %**.

Problem 5 Paper cone

$m = 4.0 \times 10^{-3} \text{ kg}$. $v_t = \sqrt{(mg/c)} = \sqrt{[(4.0 \times 10^{-3} \times 9.81)/(2.0 \times 10^{-3})]} = \sqrt{19.62} = \textbf{4.4 m/s}$.

Problem 6 Zero acceleration, nonzero speed

1. Zero acceleration means the *velocity is constant*, not zero. The object keeps moving at v_t .
2. The drag force equals the weight in magnitude, so the net force is **zero**.

Problem 7 Speed in oil

1. $v_t = mg/b = 0.981/0.50 = 1.962 \text{ m/s}$; $\tau = m/b = 0.20 \text{ s}$.
2. $t/\tau = 1.5$, so $v = 1.962(1 - e^{-1.5}) = 1.962 \times 0.7769 = \textbf{1.5 m/s}$.

Problem 8 Acceleration history

1. At $t = 0$ the speed is zero, so there is no drag and $a = g = 9.81 \text{ m/s}^2$.
2. $a = g e^{-t/\tau} = 9.81 \times e^{-1} = 9.81 \times 0.368 = 3.61 \text{ m/s}^2$.
3. As v rises, drag bv rises, so the net force $mg - bv$ falls.

Problem 9 Time to reach 90 %

1. $1 - e^{-t/\tau} = 0.90$, so $e^{-t/\tau} = 0.10$ and $t = \tau \ln 10$.
2. $t = 0.20 \times 2.303 = 0.46 \text{ s}$.

Problem 10 Reading a graph

1. The speed levels off at **2.0 m/s**.
2. 63 % of 2.0 m/s is 1.26 m/s, reached at 0.25 s, so $\tau = 0.25 \text{ s}$.
3. $b = mg/v_t = (0.040 \times 9.81)/2.0 = 0.20 \text{ kg/s}$. Check: $m/\tau = 0.040/0.25 = 0.16 \text{ kg/s}$, close given the graph reading.

Problem 11 Parachute constant

1. At constant speed $cv^2 = mg$, so $c = mg/v^2 = (75 \times 9.81)/36 = 20.4$, which rounds to **20 kg/m**.
2. $F = cv^2 = 20.4 \times 9.0 = 1.8 \times 10^2 \text{ N}$.

Problem 12 Doubling the mass

1. $v_t = mg/b$, so it doubles: factor **2**.
2. $v_t = \sqrt{(mg/c)}$, so it rises by $\sqrt{2}$: factor **1.4**.
3. Drag in the quadratic model grows as v^2 , so a speed increase of only $\sqrt{2}$ is needed to balance double the weight.

Problem 13 Spot the sign error

1. Drag opposes the downward velocity, so it acts upward and must be *negative* when down is positive.
2. **$m \, dv/dt = mg - bv$** .
3. With $+bv$ the acceleration would grow without limit as the ball speeds up: there would be no terminal speed, which is unphysical.

Problem 14 Verify the solution

1. $dv/dt = v_t(1/\tau) e^{-t/\tau} = (mg/b)(b/m) e^{-t/\tau} = g e^{-t/\tau}$.
 2. Left side: $m \, dv/dt = mg e^{-t/\tau}$. Right side: $mg - bv = mg - b(mg/b)(1 - e^{-t/\tau}) = mg e^{-t/\tau}$.
- The two sides agree**, and $v(0) = 0$.

Problem 15 Distance fallen

- $x = \int v \, dt = v_t t - v_t \tau = v_t t - v_t \tau (1 - e^{-t/\tau})$.
- At $t = \tau$: $x = v_t \tau e^{-1} = 1.5 \times 0.30 \times 0.3679 = \mathbf{0.17 \, m}$.

Problem 16 Boat coasting to rest

- $m \, dv/dt = -bv$ gives $dv/v = -(b/m) \, dt$; integrating, $\ln(v/v_0) = -bt/m$.
- $\tau = m/b = 8.0 \, \text{s}$; $v = 6.0 \, e^{-1.5} = 6.0 \times 0.2231 = \mathbf{1.3 \, m/s}$.
- $x = \int_0^\infty v \, dt = v_0 \tau = 6.0 \times 8.0 = \mathbf{48 \, m}$.

Problem 17 Skydiver with tanh

- $gt/v_t = 29.43/30 = 0.981$; $\tanh(0.981) = 0.754$; $v = \mathbf{23 \, m/s}$.
- $v = gt = \mathbf{29 \, m/s}$.
- Drag has already reduced the speed by about 7 m/s: at 23 m/s the drag is $(23/30)^2 = 59 \, \%$ of the weight.

Problem 18 Ball thrown upward

- $a = g + (c/m)v^2 = 9.81 + 2.0 \times 10^{-3} \times 625 = \mathbf{11.1 \, m/s^2}$ (downward).
- $v_t = \sqrt{g/(c/m)} = \sqrt{9.81/0.0020} = \mathbf{70 \, m/s}$.
- With $k = c/m$, separate: $v \, dv/(g + kv^2) = -dx$. Integrating from v_0 to 0: $(1/2k) \ln[(g + kv_0^2)/g] = H$. So $H = (1/2k) \ln(1 + kv_0^2/g) = 250 \times \ln(1.1274) = 250 \times 0.1199 = \mathbf{30 \, m}$.

Problem 19 Block on an oiled ramp

- Taking down the slope as positive: $m \, dv/dt = mg \sin 30^\circ - bv$.
- At v_t , $bv_t = mg \sin 30^\circ$, so $v_t = (2.0 \times 9.81 \times 0.50)/5.0 = \mathbf{2.0 \, m/s}$.
- $\tau = m/b = 2.0/5.0 = \mathbf{0.40 \, s}$.

Problem 20 Power at terminal speed

- The kinetic energy is constant, so all the energy lost by gravity each second is transferred to the air by the drag force: $P = F_D v_t = mg v_t$.
- $P = 80 \times 9.81 \times 62.6 = 4.9 \times 10^4 \, \text{W} = \mathbf{49 \, kW}$.