

Angular motion, torque and balance

AP Physics 1 (algebra-based) · Lesson

Quick answer: Torque is the turning effect of a force, $\tau = r F \sin \theta$, measured in newton metres. Rotation is described by angle in radians, angular velocity in rad/s and angular acceleration in rad/s^2 , with equations that mirror straight-line motion. An object is in static equilibrium when the net force and the net torque are both zero.

Level: AP Physics 1 (algebra-based) · **Curriculum:** AP P1 Unit 5

How do you describe rotation?

Rotation uses angle instead of distance. An object turning about a fixed axis has an angular position, angular velocity and angular acceleration, each with a straight-line partner.

Definition

Angular displacement θ is the angle turned, in radians (rad). **Angular velocity** $\omega = \Delta\theta \div \Delta t$, in rad/s. **Angular acceleration** $\alpha = \Delta\omega \div \Delta t$, in rad/s^2 . One revolution is $2\pi \text{ rad} = 360^\circ$, and $1 \text{ rpm} = 2\pi \div 60 \text{ rad/s}$.

Quantity	Straight-line motion	Rotation about a fixed axis
position change	$x \text{ (m)}$	$\theta \text{ (rad)}$
rate of change	$v = \Delta x \div \Delta t$	$\omega = \Delta\theta \div \Delta t$
acceleration	$a = \Delta v \div \Delta t$	$\alpha = \Delta\omega \div \Delta t$
constant acceleration	$v = u + at$; $x = ut + \frac{1}{2}at^2$	$\omega = \omega_0 + \alpha t$; $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
link for a point at radius r	$s = r\theta$	$v = r\omega$, $a_t = r\alpha$

Formulae

$$\omega = \omega_0 + \alpha t, \theta = \omega_0 t + \frac{1}{2} \alpha t^2, \omega^2 = \omega_0^2 + 2 \alpha \theta$$

ω_0 = starting angular velocity (rad/s), α = constant angular acceleration (rad/s^2), t = time (s), θ = angle (rad).

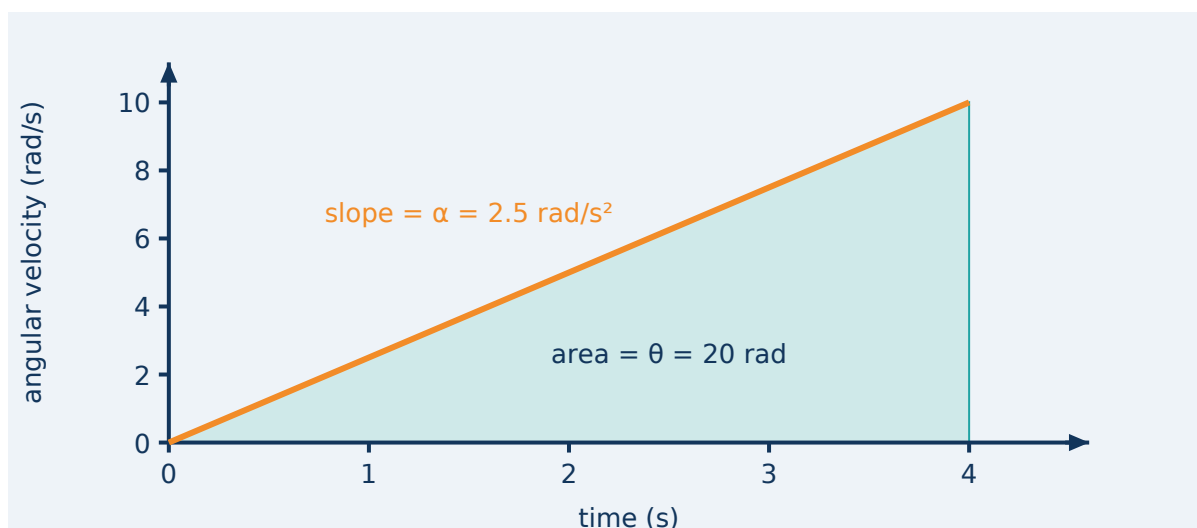
Worked example

A ceiling fan in Nairobi starts from rest and speeds up at 2.5 rad/s^2 for 4.0 s. Its blade tip is 0.60 m from the axis.

$$\omega = 0 + 2.5 \times 4.0 = \mathbf{10 \text{ rad/s.}}$$

$$\theta = \frac{1}{2} \times 2.5 \times 4.0^2 = \mathbf{20 \text{ rad}}, \text{ which is } 20 \div 2\pi = 3.2 \text{ revolutions.}$$

$$\text{Tip speed } v = r\omega = 0.60 \times 10 = \mathbf{6.0 \text{ m/s.}}$$



The area under an ω - t graph is the angle turned, just as the area under a v - t graph is distance.

What is torque and how do you calculate it?

A force changes rotation only through its turning effect. That effect grows with the size of the force and with how far from the axis it acts. Only the part of the force at right angles to the line from the axis counts.

Definition

Torque is the turning effect of a force about an axis: $\tau = r F \sin \theta$, where r is the distance from the axis to the point of application and θ is the angle between r and F . The perpendicular distance from the axis to the line of the force is the **lever arm**, $r \sin \theta$.

Formula

$$\tau = r F \sin \theta$$

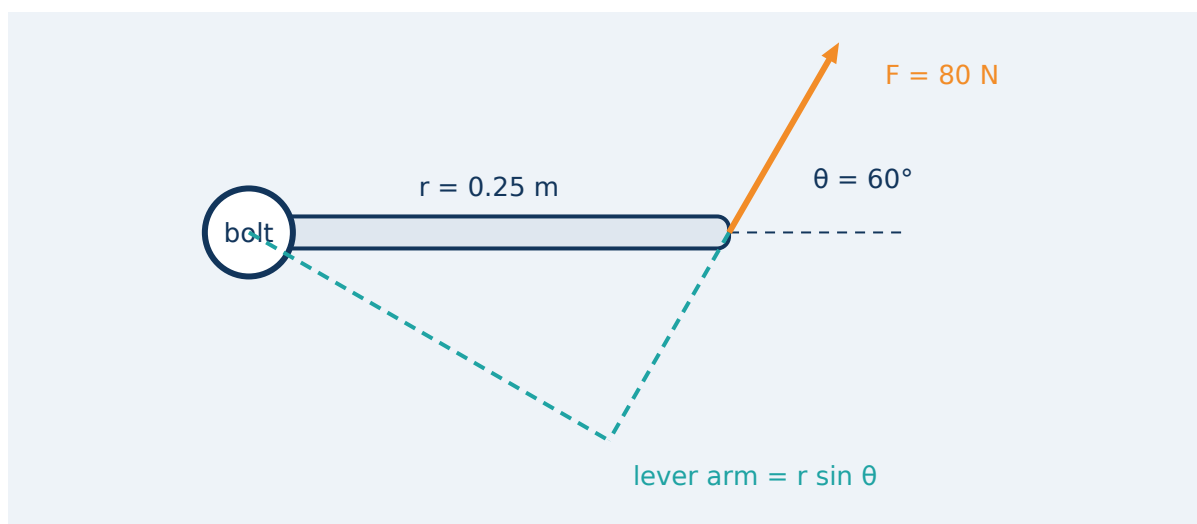
τ = torque (N m), r = distance from the axis (m), F = force (N), θ = angle between r and F . Take anticlockwise torque as positive and clockwise as negative.

Worked example

Wanjiru pulls on a spanner with $F = 80$ N. The force is 0.25 m from the bolt and makes an angle of 60° with the handle.

$$\tau = r F \sin \theta = 0.25 \times 80 \times \sin 60^\circ = 0.25 \times 80 \times 0.866 = \mathbf{17 \text{ N m.}}$$

If she pulled at 90° she would get 20 N m, the largest possible with this force.



Torque depends on the force, the distance from the bolt and the angle between them.

When is an object in static equilibrium?

An object that is not accelerating and not rotating is in static equilibrium. Two conditions must hold together, and each can be applied about any point you choose. Choosing the hinge as the axis removes the unknown hinge force from the torque equation.

Key law

Static equilibrium: the net force is zero ($\Sigma F = 0$ in each direction) and the net torque about any axis is zero ($\Sigma \tau = 0$). The weight of a uniform object acts at its centre.

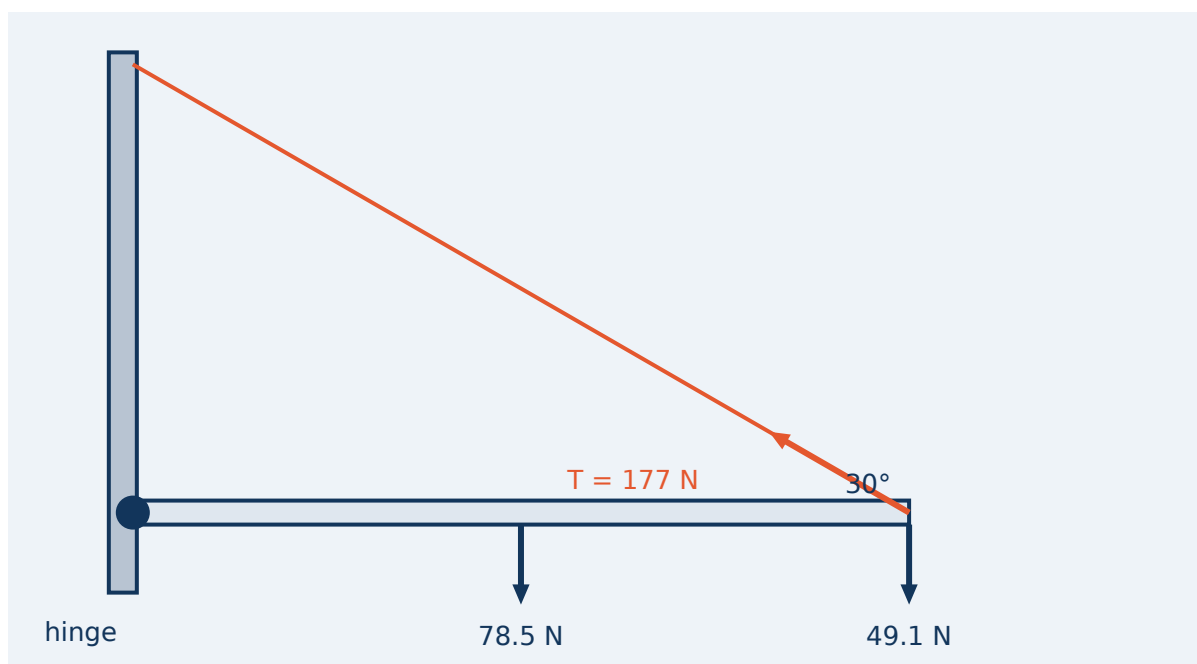
Worked example

A uniform pole of mass 8.0 kg and length 2.0 m is hinged to a wall. A cable at 30° above the pole holds the free end, and a 5.0 kg sign hangs from that end. Take torques about the hinge.

$$T \times (2.0 \sin 30^\circ) = (8.0 \times 9.81) \times 1.0 + (5.0 \times 9.81) \times 2.0 = 78.5 + 98.1 = 176.6 \text{ N m.}$$

$$T = 176.6 \div 1.0 = \mathbf{177 \text{ N.}}$$

Hinge force: horizontal = $T \cos 30^\circ = 153 \text{ N}$ away from the wall; vertical = $127.5 - 88.3 = \mathbf{39 \text{ N}}$ upward.



The weights, the cable force and the hinge force balance both forces and torques.

Common mistake

Do not multiply a force by the full distance to the axis when the force is not perpendicular. Use either $r F \sin \theta$, or the perpendicular component of the force times r . Check that the torques you add have the right signs.

Lina's tip

Choose the axis where the most unknown forces act, usually the hinge. Those forces have zero lever arm, so they vanish from the torque equation.

Key points to remember

- Angle in radians: $2\pi \text{ rad} = 360^\circ$; $\omega = \Delta\theta \div \Delta t$; $\alpha = \Delta\omega \div \Delta t$.
- Constant α : $\omega = \omega_0 + \alpha t$, $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, $\omega^2 = \omega_0^2 + 2\alpha\theta$.
- For a point at radius r : $s = r\theta$, $v = r\omega$, $a_t = r\alpha$.
- Torque $\tau = rF \sin \theta$ (N m); anticlockwise positive.
- Static equilibrium: $\Sigma F = 0$ and $\Sigma \tau = 0$ about any axis.

Angular motion, torque and balance: frequently asked questions

What is the difference between torque and force?

Force causes linear acceleration, while torque causes angular acceleration about an axis. Torque depends on the size of the force, where it acts and its direction relative to the axis.

Why are angles measured in radians in rotation equations?

Radians make arc length equal to $r\theta$, so $v = r\omega$ and $a = r\alpha$ hold without conversion factors.

With degrees these links would need an extra factor of $\pi \div 180$.

How do you find the direction of a torque?

Imagine the force turning the object about the axis. Anticlockwise rotation counts as positive and clockwise as negative. The sign lets you add torques correctly in an equilibrium equation.

Where does the weight of a uniform beam act?

At its geometric centre. Torque calculations treat the entire weight as a single force there, at half the length from either end.

Why can you choose any axis in an equilibrium problem?

If the net torque is zero about one point and the net force is zero, it is zero about every point. A good choice removes unknown forces and makes the algebra shorter.