

Angular motion, torque and balance

AP Physics 1 (algebra-based) · Worksheet answers

Every answer gives the formula, the substitution and the result with its unit. Take $g = 9.81 \text{ N/kg}$ where weights appear. If your result differs only in the last digit, check the rounding. For written problems, compare the physics reasons in the model answer rather than the exact wording.

Level: AP Physics 1 (algebra-based) · **Curriculum:** AP P1 Unit 5

Problem 1 Angle conversions

- $90^\circ \times \pi \div 180^\circ = \pi \div 2 \text{ rad}$ (1.57 rad).
- $2.5 \times 2\pi = 5\pi \text{ rad} = 15.7 \text{ rad}$.
- $(3\pi \div 2) \times 180^\circ \div \pi = 270^\circ$.

Problem 2 Potter's wheel speed

- $\omega = 45 \times 2\pi \div 60 = 4.7 \text{ rad/s}$ (4.712).
- $\theta = \omega t = 4.712 \times 10 = 47 \text{ rad}$, which is $47.1 \div 2\pi = 7.5 \text{ revolutions}$ (45 rpm for 10 s is 7.5 rev).

Problem 3 Motorbike tyre

- $v = r \omega = 0.32 \times 40 = 12.8 \text{ m/s}$.
- $s = v t = 12.8 \times 5.0 = 64 \text{ m}$. (Check: $\theta = 200 \text{ rad}$ and $s = r\theta = 0.32 \times 200 = 64 \text{ m}$.)

Problem 4 Pushing a door

- $\tau = r F = 0.80 \times 30 = 24 \text{ N m}$.
- $\tau = 0.20 \times 30 = 6.0 \text{ N m}$.
- The same force gives a larger torque at a larger distance, so a smaller force opens the door.

Problem 5 Signs of torque

- +5.0 N m, -3.0 N m and +1.0 N m (each is $0.50 \times F$).
- Net = $5.0 - 3.0 + 1.0 = +3.0 \text{ N m}$, so the wheel tends to turn **anticlockwise**.

Problem 6 Record deck spin-up

- $\omega = 33.33 \times 2\pi \div 60 = 3.49 \text{ rad/s}$.
- $\alpha = \Delta\omega \div \Delta t = 3.49 \div 2.0 = 1.75 \text{ rad/s}^2$.

Problem 7 Seesaw in the park

1. For balance the clockwise and anticlockwise torques are equal: $36 \times 9.81 \times 1.8 = m \times 9.81 \times 1.2$.
2. $m = 36 \times 1.8 \div 1.2 = \mathbf{54 \text{ kg}}$. The gravitational field strength cancels.

Problem 8 Bicycle wheel slowing

1. $t = (0 - 24) \div (-3.0) = \mathbf{8.0 \text{ s}}$.
2. $\theta = \omega_0 t + \frac{1}{2} \alpha t^2 = 24 \times 8.0 - \frac{1}{2} \times 3.0 \times 64 = 192 - 96 = \mathbf{96 \text{ rad}} = 96 \div 2\pi = \mathbf{15 \text{ revolutions}}$.

Problem 9 Reading an angular velocity graph

1. 0-3 s: $(12 - 0) \div 3.0 = \mathbf{+4.0 \text{ rad/s}^2}$. 3-7 s: $\mathbf{0}$. 7-9 s: $(0 - 12) \div 2.0 = \mathbf{-6.0 \text{ rad/s}^2}$.
2. Area = $\frac{1}{2} \times 3.0 \times 12 + 4.0 \times 12 + \frac{1}{2} \times 2.0 \times 12 = 18 + 48 + 12 = \mathbf{78 \text{ rad}}$.

Problem 10 Spanner at an angle

1. $\tau = 0.30 \times 60 \times \sin 40^\circ = 18 \times 0.643 = \mathbf{11.6 \text{ N m}}$.
2. The largest torque is at 90° : $\tau = 0.30 \times 60 = \mathbf{18 \text{ N m}}$.

Problem 11 Balancing a metre rule

1. At the centre of the rule, the 50 cm mark.
2. Centre: $0.50 - 0.40 = 0.10 \text{ m}$ on one side; hung mass: $0.40 - 0.05 = 0.35 \text{ m}$ on the other.
3. $m \times 0.35 = 0.15 \times 0.10$, so $m = 0.015 \div 0.35 = \mathbf{0.043 \text{ kg}}$ (43 g).

Problem 12 Bicycle chain drive

1. Front $\omega = 60 \times 2\pi \div 60 = 6.28 \text{ rad/s}$; chain speed $v = r\omega = 0.10 \times 6.28 = \mathbf{0.63 \text{ m/s}}$.
2. $\omega_{\text{rear}} = v \div r = 0.628 \div 0.040 = \mathbf{16 \text{ rad/s}}$.
3. Bicycle speed = $0.33 \times 15.7 = \mathbf{5.2 \text{ m/s}}$.

Problem 13 Net torque on a light rod

1. Up at 1.0 m: $+15 \text{ N m}$ (anticlockwise). Down at 0.40 m: -4.0 N m . Net = $\mathbf{+11 \text{ N m}}$.
2. $F \times 1.1 = 11$, so $F = \mathbf{10 \text{ N}}$.

Problem 14 Scaffold plank

1. $N_2 \times 3.0 = 24 \times 9.81 \times 1.5 + 60 \times 9.81 \times 1.0 = 353.2 + 588.6 = 941.8$, so $N_2 = \mathbf{314 \text{ N}}$.
2. $N_1 + N_2 = 84 \times 9.81 = 824.0 \text{ N}$, so $N_1 = 824.0 - 313.9 = \mathbf{510 \text{ N}}$.

Problem 15 Long handles

1. $F = \tau \div r = 40 \div 0.40 = \mathbf{100\ N}$.
2. $F = 40 \div 0.25 = \mathbf{160\ N}$.
3. For the same torque, a longer lever arm needs a smaller force, so the nut is easier to loosen.

Problem 16 Spinning up and doubling the time

1. $\theta = \frac{1}{2} \times 0.50 \times 6.0^2 = \mathbf{9.0\ rad}$; $\omega = 0.50 \times 6.0 = \mathbf{3.0\ rad/s}$.
2. $a_t = r\alpha = 0.40 \times 0.50 = \mathbf{0.20\ m/s^2}$; $v = r\omega = 0.40 \times 3.0 = \mathbf{1.2\ m/s}$.
3. At 12 s: $\theta = \frac{1}{2} \times 0.50 \times 144 = 36\ rad$, four times the angle. **The claim is wrong:** from rest, θ is proportional to t^2 , so doubling t quadruples θ .

Problem 17 Verifying torque with a rule

1. Pivot the rule at its centre so its own weight has no torque. Hang a fixed load M at a distance r from the pivot. Hang a smaller mass m on the other side and slide it until the rule is horizontal; record its distance d . Repeat for several values of r .
2. If torque balance holds, $M g r = m g d$, so a graph of d against r is a **straight line through the origin with gradient $M \div m$** . Compare the gradient with the ratio of masses.
3. The pivot may not be exactly at the centre. Balance the unloaded rule first and re-position the pivot, or measure the rule's mass and include its torque.

Problem 18 Hinged beam with a cable

1. $T \sin 40^\circ \times 2.0 = 12 \times 9.81 \times 1.5 + 8.0 \times 9.81 \times 3.0 = 176.6 + 235.4 = 412.0$, so $T = 412.0 \div 1.2856 = \mathbf{320\ N}$.
2. Horizontal: $H = T \cos 40^\circ = 320.5 \times 0.766 = \mathbf{245\ N}$ away from the wall. Vertical: weights 196.2 N, cable lifts $T \sin 40^\circ = 206.0\ N$, so the hinge must push **9.8 N downward**.

Problem 19 Analysing wheel data

1. For constant α from rest, $\theta = \frac{1}{2} \alpha t^2$, which has the form $y = m x$ with $x = t^2$.
2. t^2 values are 1.0 and 16.0. Gradient = $(24.1 - 1.5) \div (16.0 - 1.0) = 1.507\ rad/s^2$, so $\alpha = 2 \times 1.507 = \mathbf{3.0\ rad/s^2}$.
3. $\omega = \alpha t = 3.0 \times 4.0 = \mathbf{12\ rad/s}$. Average = $24.1 \div 4.0 = 6.0\ rad/s$, half the final speed, as expected for uniform acceleration from rest.

Problem 20 Equal forces, no equilibrium

1. Net force = $12 - 12 = \mathbf{0}$. Each force has a lever arm of 0.15 m about the centre and both torques turn the same way: $2 \times 12 \times 0.15 = \mathbf{3.6\ N\ m}$.
2. The stick is not in equilibrium: although the net force is zero, the net torque is not, so it starts to spin without moving as a whole. About one end the torque is $12 \times 0.30 = 3.6\ N\ m$ from the far force alone, the same value, because for zero net force the net torque does not

depend on the axis chosen.