

Checking units, uncertainties and vector components

Physics grades 11-12 (A Level · IB · Class 11-12) · Lesson

Quick answer: Homogeneity means every term of an equation has the same base units, which lets you test formulae. Uncertainty states how far a reading may be wrong: add absolute uncertainties for sums and percentage uncertainties for products. A vector is resolved into components with $F \cos \theta$ and $F \sin \theta$, then added by components.

Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** A Level 1, IB A

Why do units have to match in every equation?

Physics lets you add or compare only quantities that have the same units. Every unit in the SI system is built from seven base units; in mechanics you meet the metre (m), kilogram (kg) and second (s), together with the ampere (A) and kelvin (K) later on. All other units, the *derived units*, are combinations of these.

Definition

An equation is **homogeneous** when every term has the same base units on both sides. A homogeneous equation can still be wrong, but an equation that is not homogeneous must be wrong.

Quantity	Derived unit	In base units
Force	newton (N)	kg m s^{-2}
Pressure	pascal (Pa)	$\text{kg m}^{-1} \text{s}^{-2}$
Energy	joule (J)	$\text{kg m}^2 \text{s}^{-2}$
Power	watt (W)	$\text{kg m}^2 \text{s}^{-3}$
Frequency	hertz (Hz)	s^{-1}

Worked example 1

Test $s = ut + \frac{1}{2}at^2$. Base units: $u t$ gives $(\text{m s}^{-1})(\text{s}) = \text{m}$, and $a t^2$ gives $(\text{m s}^{-2})(\text{s}^2) = \text{m}$. Both terms are lengths, so the equation is homogeneous. Now test $v = u + at^2$: the first two terms are in m s^{-1} , but at^2 is in m , so the equation **cannot be correct**.

How do you work out an uncertainty?

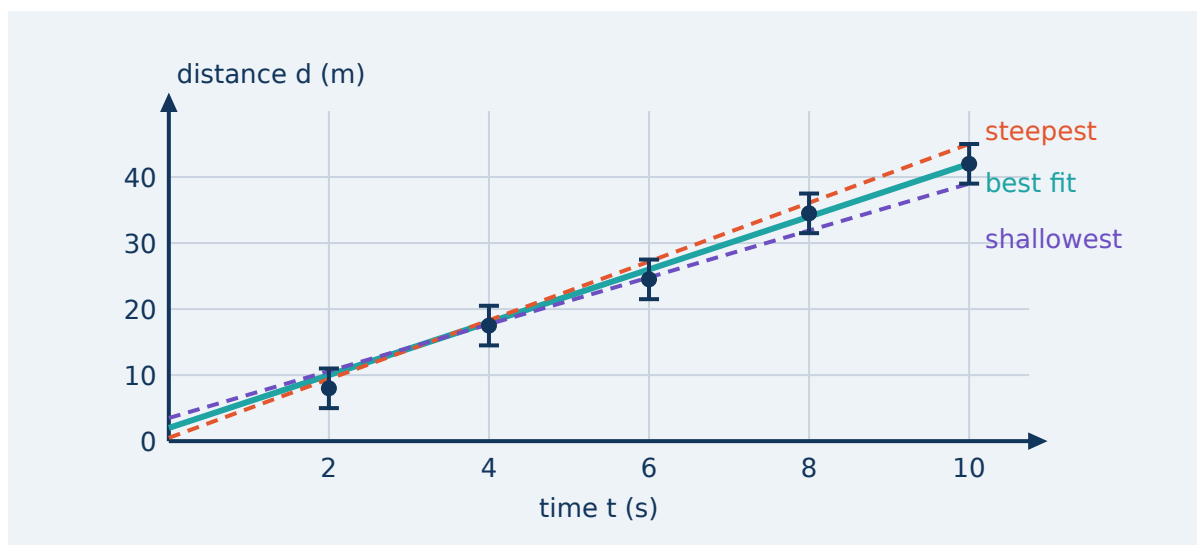
Every measurement has an **absolute uncertainty**, written $\pm \Delta x$, in the same unit as the reading. For one reading it is usually half the smallest division, or the full division when you

must judge the start and the end. For repeated readings, take the mean and use half the range (largest minus smallest) as the uncertainty. The **percentage uncertainty** compares the uncertainty with the size of the reading.

Formulae

Percentage uncertainty = $(\Delta x \div x) \times 100 \%$. Here Δx is the absolute uncertainty and x is the measured value, both in the same unit.

Sum or difference: add the *absolute* uncertainties. Product or quotient: add the *percentage* uncertainties. Power x^n : multiply the percentage uncertainty by n .



The steepest and shallowest lines through the error bars give the uncertainty in the gradient: $(\text{steepest} - \text{shallowest}) \div 2$.

Worked example 2

A plate measures 12.0 ± 0.1 cm by 8.0 ± 0.1 cm. Percentage uncertainties: $0.1 \div 12.0 = 0.83 \%$ and $0.1 \div 8.0 = 1.25 \%$. The area is $12.0 \times 8.0 = 96.0$ cm² with percentage uncertainty $0.83 + 1.25 = 2.08 \%$. Absolute uncertainty = $0.0208 \times 96.0 = 2.0$ cm². Area = **96 ± 2 cm²**.

Common mistake

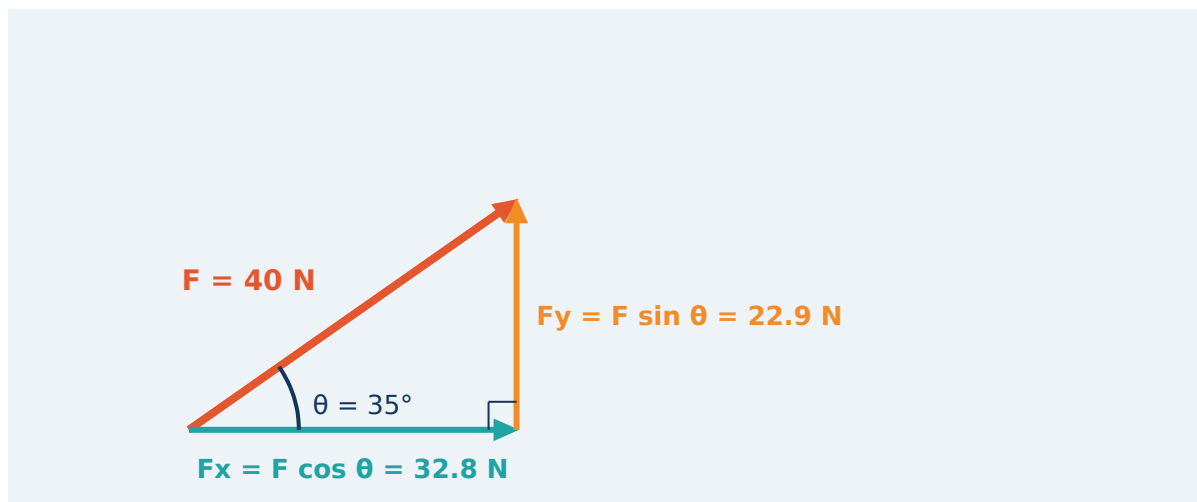
Students add percentage uncertainties for a sum, or add absolute uncertainties for a product. Remember: adding or subtracting quantities means absolute uncertainties; multiplying or dividing means percentages. Subtracting two close readings makes the percentage uncertainty much larger.

What is the difference between a scalar and a vector?

A **scalar** has magnitude only, while a **vector** has magnitude and direction. Mass, time, speed, distance, energy and temperature are scalars. Displacement, velocity, acceleration, force and momentum are vectors, so they must be added with their directions.

How do you resolve a vector into components?

Any vector can be replaced by two perpendicular components that together have the same effect. For a vector F at an angle θ above the horizontal, the horizontal component is $F \cos \theta$ and the vertical component is $F \sin \theta$. Always sketch the right-angled triangle first, because the angle you are given decides which function goes with which component.



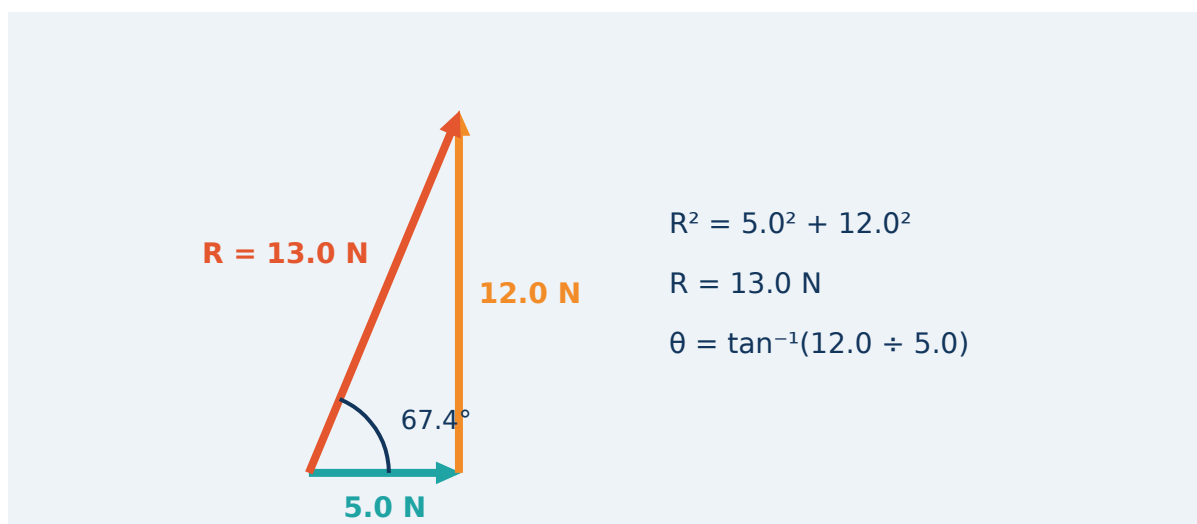
Resolving a 40 N force into perpendicular components.

Worked example 3

A rope pulls a cart with 40 N at 35° above the horizontal. $F_x = 40 \cos 35^\circ = 40 \times 0.819 = 32.8 \text{ N}$. $F_y = 40 \sin 35^\circ = 40 \times 0.574 = 22.9 \text{ N}$. Check: $\sqrt{(32.8^2 + 22.9^2)} = 40.0 \text{ N}$. The horizontal component is **32.8 N** and the vertical component is **22.9 N**.

How do you add vectors?

Place the vectors head to tail; the resultant runs from the first tail to the last head. For two perpendicular vectors, use Pythagoras for the size and the tangent for the direction. For any number of vectors, add all the x-components, add all the y-components, then combine the two totals.



Head-to-tail addition of two perpendicular vectors.

$$R^2 = 5.0^2 + 12.0^2$$

$$R = 13.0 \text{ N}$$

$$\theta = \tan^{-1}(12.0 \div 5.0)$$

Lina's tip

Before you calculate, write the unit of the answer you expect. If your final line gives joules when you wanted newtons, you know straight away that a term has been left out or inverted.

Key points to remember

- Base units: m, kg, s, A, K; newton = kg m s^{-2}
- Percentage uncertainty = $(\Delta x \div x) \times 100 \%$
- Sum or difference: add absolute uncertainties; product or quotient: add percentages
- Component next to the angle: $F \cos \theta$; opposite the angle: $F \sin \theta$
- Resultant of perpendicular vectors: $R = \sqrt{x^2 + y^2}$, $\tan \theta = y \div x$

Checking units, uncertainties and vector components: frequently asked questions

What does homogeneous mean in physics?

An equation is homogeneous when all its terms reduce to the same base units. It is a quick test: if the units on the two sides differ, the equation is wrong. Matching units do not prove it is right, because numerical factors such as $\frac{1}{2}$ cannot be checked this way.

How do you find a percentage uncertainty?

Divide the absolute uncertainty by the measured value and multiply by 100. For a reading of $24.0 \text{ cm} \pm 0.2 \text{ cm}$ the result is 0.83 %. Keep one or two significant figures only.

What is the uncertainty of a power such as r^3 ?

Multiply the percentage uncertainty of r by the power. If r has 2 % uncertainty, r^3 has 6 %. This works because r^3 is $r \times r \times r$ and the percentage uncertainties add.

How do you resolve a force at an angle?

Draw a right-angled triangle with the force as the hypotenuse. The component next to the angle is $F \cos \theta$ and the component opposite is $F \sin \theta$. Check by confirming that the components give back the original force.

How do you find the uncertainty in a gradient from a graph?

Draw the best-fit line, then the steepest and shallowest lines that still pass through all the error bars. The uncertainty in the gradient is half the difference between the steepest and shallowest gradients.