

Waves that add: fringes and grating spectra

Physics grades 11-12 (A Level · IB · Class 11-12) · Lesson

Quick answer: Superposition means waves add their displacements where they meet. Constructive interference needs a path difference of $n\lambda$; destructive needs $(n + \frac{1}{2})\lambda$. In a double slit the bright fringes are $w = \lambda D/s$ apart, and a diffraction grating gives sharp orders where $d \sin \theta = n\lambda$.

Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** IB C.3

What is the principle of superposition?

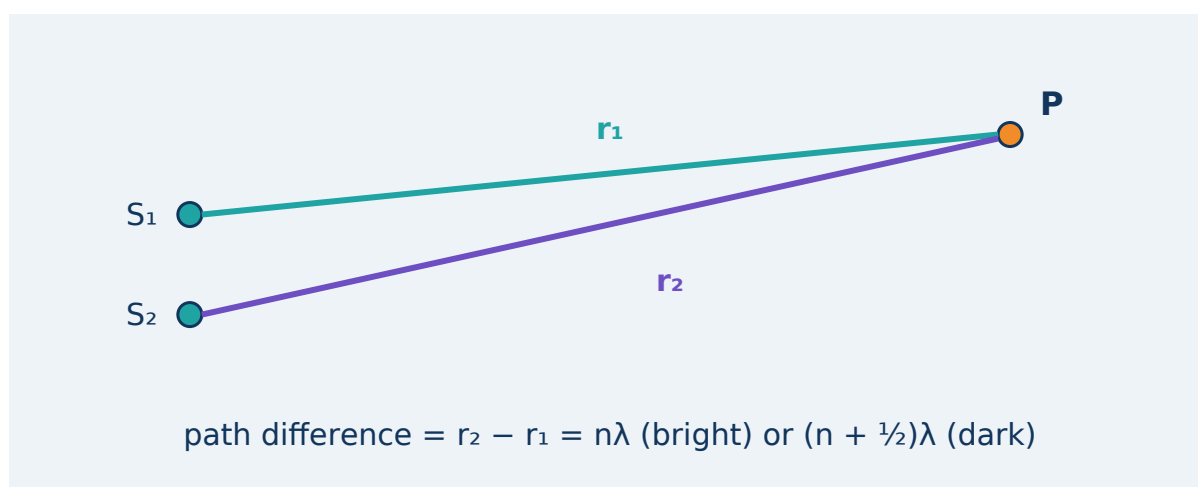
Definition

The **principle of superposition** states that when two or more waves meet at a point, the resultant displacement is the vector sum of the displacements of the individual waves at that point.

Two crests arriving together make a larger crest; a crest meeting a trough of equal size cancels. This adding of displacements is called *interference*. Clear, stable interference needs **coherent** sources: sources with a constant phase difference and the same frequency. A laser and a pair of slits lit by one beam are coherent; two separate torches are not.

When do waves interfere constructively or destructively?

It depends on the **path difference**, the extra distance one wave travels compared with the other. If the path difference is a whole number of wavelengths, the waves arrive in step. If it is an odd number of half wavelengths, they arrive in antiphase.



Path difference $r_2 - r_1$ between two coherent sources and a point P.

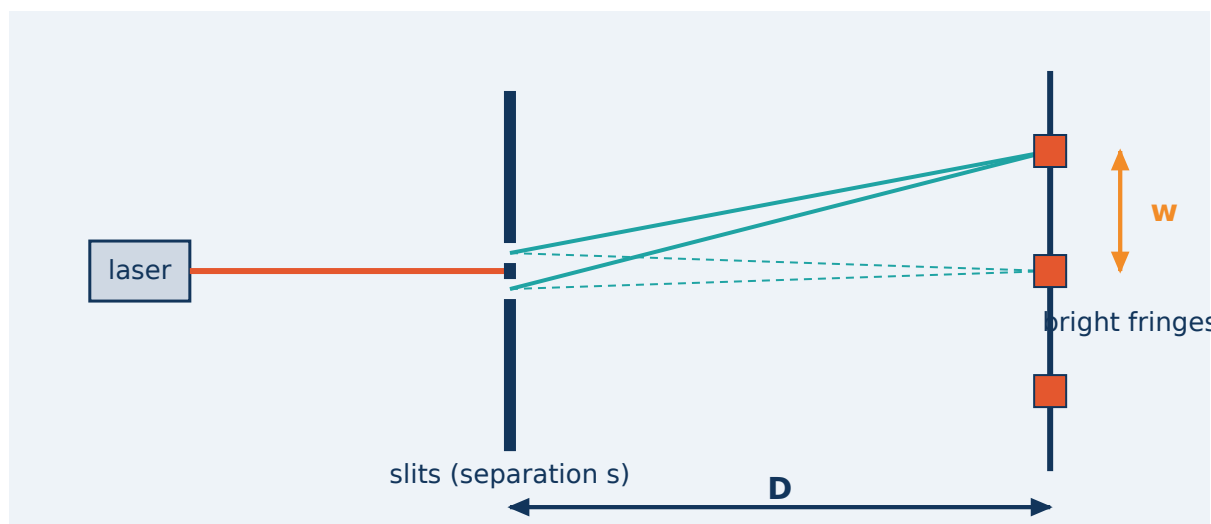
Key law

Constructive interference: path difference = $n\lambda$ ($n = 0, 1, 2, \dots$). Destructive interference:

path difference = $(n + \frac{1}{2})\lambda$. The phase difference is $2\pi \times (\text{path difference}) \div \lambda$.

How does a double-slit experiment work?

Light from one laser passes through two narrow slits a distance s apart. Each slit acts as a new source (by diffraction), and the two beams overlap on a distant screen at distance D . Where the path difference is $n\lambda$, you see a bright fringe; where it is $(n + \frac{1}{2})\lambda$, a dark fringe. For small angles the fringes are equally spaced.



Double-slit interference pattern. The fringe spacing w is the distance between neighbouring bright fringes.

Double-slit formula

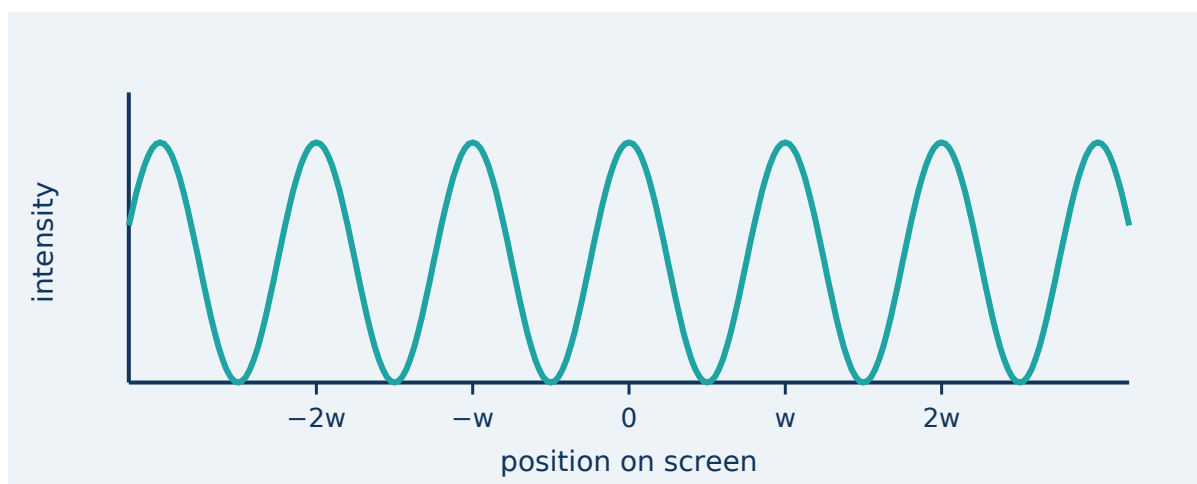
$$w = \lambda D / s$$

w is the fringe spacing (m), λ the wavelength (m), D the slit-to-screen distance (m), s the slit separation (m). Valid when s is much smaller than D .

Worked example 1

A red laser of wavelength 650 nm shines on slits 0.30 mm apart. The screen is 2.00 m away. Calculate the fringe spacing.

$$w = \lambda D / s = (650 \times 10^{-9} \times 2.00) \div (0.30 \times 10^{-3}) = \mathbf{4.33 \text{ mm}}.$$

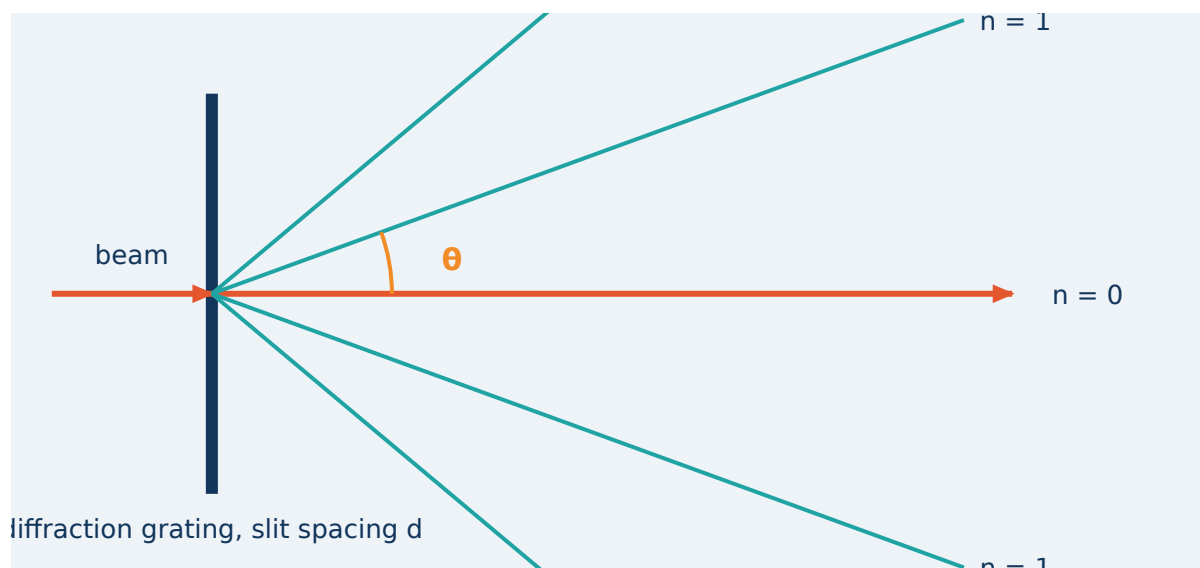


Ideal double-slit intensity pattern. Bright fringes are equally spaced and equally bright.

Change to the set-up	Effect on fringe spacing w
Increase the wavelength λ (red instead of blue)	w increases
Increase the screen distance D	w increases
Increase the slit separation s	w decreases
Immerse the apparatus in water	w decreases (λ is shorter)

What does a diffraction grating do?

A diffraction grating has hundreds of equally spaced slits per millimetre. Light from all the slits reinforces only in particular directions where the path difference between neighbouring slits is a whole number of wavelengths. These directions give sharp, bright spots called *orders*.



Orders from a diffraction grating. The condition is $d \sin \theta = n\lambda$.

Grating equation

$$d \sin \theta = n\lambda$$

d is the distance between adjacent slits (m), θ the angle of the order to the straight-through beam, n the order (0, 1, 2...), λ the wavelength (m). For N lines per metre, $d = 1 \div$

N.

Worked example 2

A grating has 500 lines per mm. Green laser light of 532 nm hits it at right angles. Find the angle of each order and the highest order.

$d = 1 \div 500\,000 = 2.00 \times 10^{-6}$ m. First order: $\sin \theta = \lambda/d = 0.266$, so $\theta = 15.4^\circ$. Second order: $\sin \theta = 0.532$, $\theta = 32.1^\circ$. Third order: $\sin \theta = 0.798$, $\theta = 52.9^\circ$. Highest order: $d/\lambda = 3.76$, so $\sin \theta$ cannot exceed 1 and $n_{\max} = 3$.

What is diffraction through a single gap?

Waves spread out when they pass through a gap or round an obstacle. The spreading is largest when the gap width is similar to or smaller than the wavelength. For a single slit of width a , the first minimum lies at $\sin \theta = \lambda/a$, so a narrower slit gives a wider central maximum. Sound (λ of about 1 m) spreads round a doorway, but light (λ of about 500 nm) hardly seems to bend at all.

Common mistake

Students measure the distance from the centre to the first dark fringe and call it w . Fringe spacing is the distance between *neighbouring bright* fringes (or neighbouring dark fringes). Measure across many fringes and divide.

Lina's tip

To reduce error, measure across ten fringes and divide by ten, not one. The same trick works for a grating when you measure the distance between the $n = -2$ and $n = +2$ spots.

Key points to remember

- Superposition: the resultant displacement is the vector sum of the individual displacements.
- Bright fringes: path difference = $n\lambda$. Dark fringes: path difference = $(n + \frac{1}{2})\lambda$.
- Double slit: fringe spacing $w = \lambda D/s$.
- Diffraction grating: $d \sin \theta = n\lambda$, with $d = 1 \div (\text{lines per metre})$.
- Coherent sources have the same frequency and a constant phase difference.

Waves that add: fringes and grating spectra: frequently asked questions

What is the difference between interference and diffraction?

Diffraction is the spreading of a wave as it passes through a gap or past an edge. Interference is the combination of two or more waves by superposition. A double-slit pattern needs both: diffraction at each slit, then interference between the two beams.

Why must the sources be coherent?

If the phase difference between sources changes randomly, the bright and dark positions move too quickly to see and the average is uniform brightness. A constant phase difference gives a steady pattern.

Why does a grating give sharper lines than a double slit?

With many slits, the waves cancel in nearly every direction except the exact angles where all of them are in step. The bright maxima become narrow and intense.

How do you find the maximum order of a grating?

Set $\sin \theta = 1$ in $d \sin \theta = n\lambda$, giving $n = d/\lambda$. Take the whole number below this value, because the angle cannot reach 90° exactly.

What happens to a double-slit pattern in white light?

The central fringe is white because all colours have zero path difference there. Other fringes are spread into spectra with violet nearest the centre and red furthest, since red has the longest wavelength.