

Work, kinetic energy and the net-work theorem

AP Physics 1 (algebra-based) · Lesson

Quick answer: Work is the energy a force transfers when it acts through a displacement: $W = F d \cos\theta$, in joules. Kinetic energy is $K = \frac{1}{2} m v^2$. The work-energy theorem states that the net work on an object equals its change in kinetic energy, $W_{\text{net}} = \Delta K$.

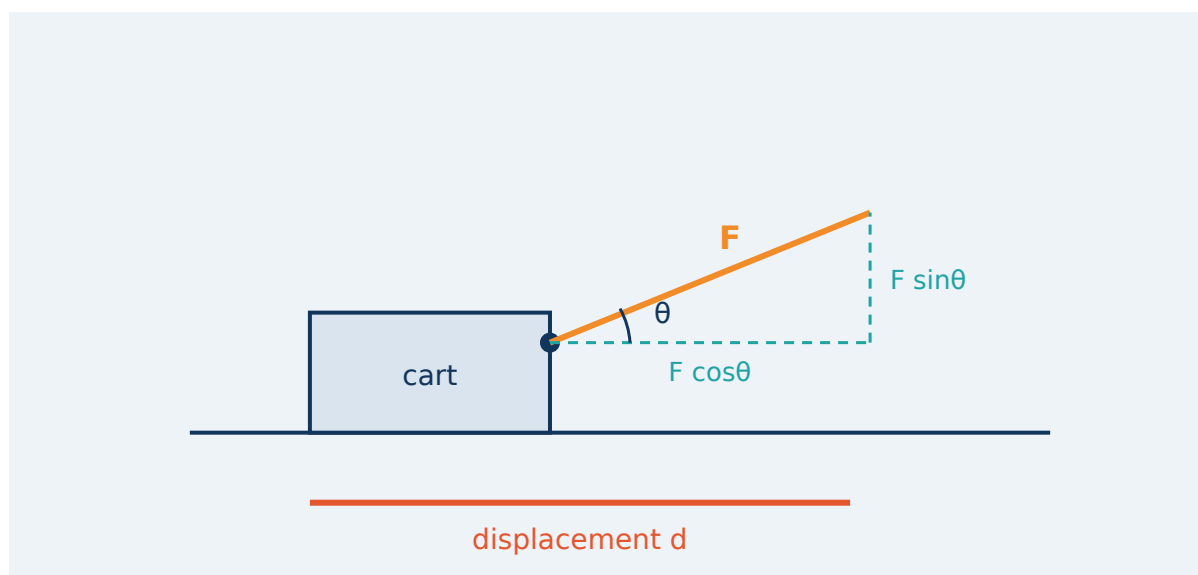
Level: AP Physics 1 (algebra-based) · **Curriculum:** AP P1 Unit 3

What is work in physics?

In everyday speech, work means effort. In physics, a force does work only when the object it acts on is displaced, and only the part of the force along the displacement counts. Holding a heavy bag still takes effort but involves no displacement, so no work is done on the bag.

Definition

Work is the energy transferred to or from an object by a force acting through a displacement. For a constant force, $W = F d \cos\theta$, where θ is the angle between the force and the displacement. Work is a scalar quantity and is measured in joules (J), with $1 \text{ J} = 1 \text{ N m}$.



Only the component of the force along the displacement, $F \cos \theta$, does work.

How do you find whether work is positive or negative?

Compare the direction of each force with the direction of motion. A force that has a component along the motion transfers energy to the object. A force with a component opposite to the motion transfers energy away from it.

Angle between force and displacement	Sign of the work	Example
0° (same direction)	positive: energy added to the object	push of the engine on an accelerating bus
Between 0° and 90°	positive, smaller than $F d$	rope pulling a sled at an angle
90° (perpendicular)	zero	normal force on a rolling cart; centripetal force on a satellite
180° (opposite direction)	negative: energy removed	friction on a sliding crate

Worked example 1

Diego in Texas pulls a loaded wagon 15 m along a level path with a handle force of 60 N at 30° above the horizontal. **Find the work done by the handle.**

$$W = F d \cos\theta = 60 \times 15 \times \cos 30^\circ = 60 \times 15 \times 0.866 = \mathbf{7.8 \times 10^2 \text{ J}}.$$

What is the work-energy theorem?

Formula

$K = \frac{1}{2} m v^2$ and $W_{\text{net}} = \Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$. Here K is kinetic energy (J), m is mass (kg), v is speed (m/s) and W_{net} is the sum of the work done by every force (J).

Key law

The net work done on an object equals the change in its kinetic energy. Positive net work speeds the object up, negative net work slows it down and zero net work leaves its speed unchanged.

The theorem works even when the forces vary or the path is curved, because it only needs the total work. Add the signed work of each force, or find the net force first and then multiply by the displacement; both routes give the same answer.

Worked example 2

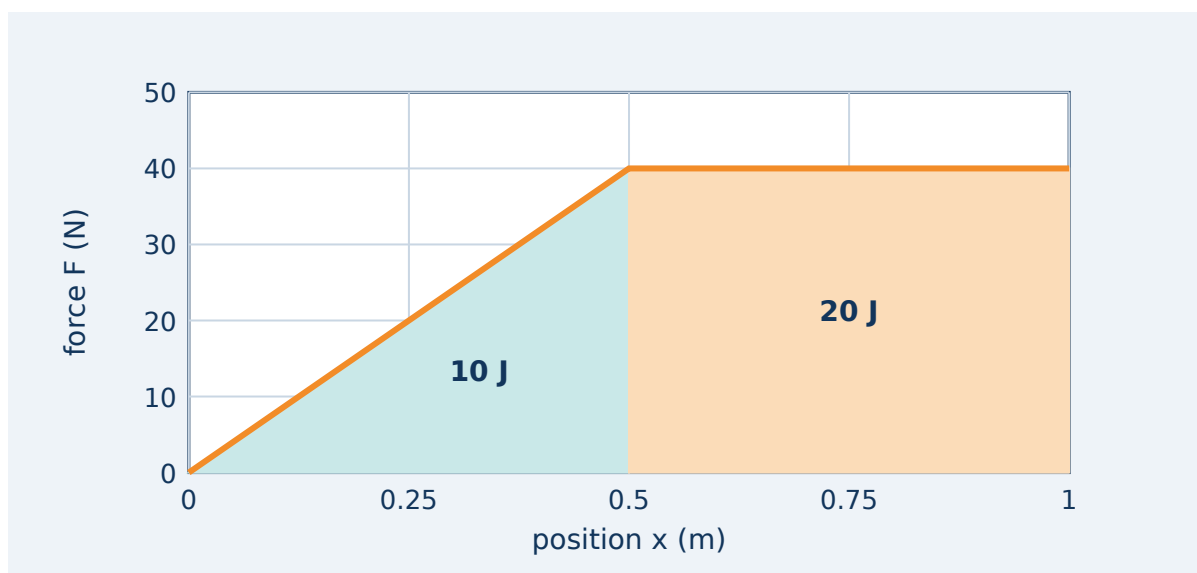
Kofi, mass 70 kg, accelerates from rest to 8.0 m/s over a straight 20 m sprint. **Find the net work done on him and the average net force.**

$$W_{\text{net}} = \Delta K = \frac{1}{2} m v^2 - 0 = \frac{1}{2} \times 70 \times 8.0^2 = 2240 \text{ J}.$$

$$F_{\text{net}} = W_{\text{net}} \div d = 2240 \div 20 = \mathbf{112 \text{ N}} \text{ (and } W_{\text{net}} = \mathbf{2.2 \times 10^3 \text{ J}}).$$

How do you find work when the force changes?

When the force is not constant, split the motion into small steps in which it is almost constant. The sum of all the pieces $F \Delta x$ is the area under the force-position graph, with areas above the axis counted positive and areas below negative.



Work equals the area under the force-position graph: $10 \text{ J} + 20 \text{ J} = 30 \text{ J}$.

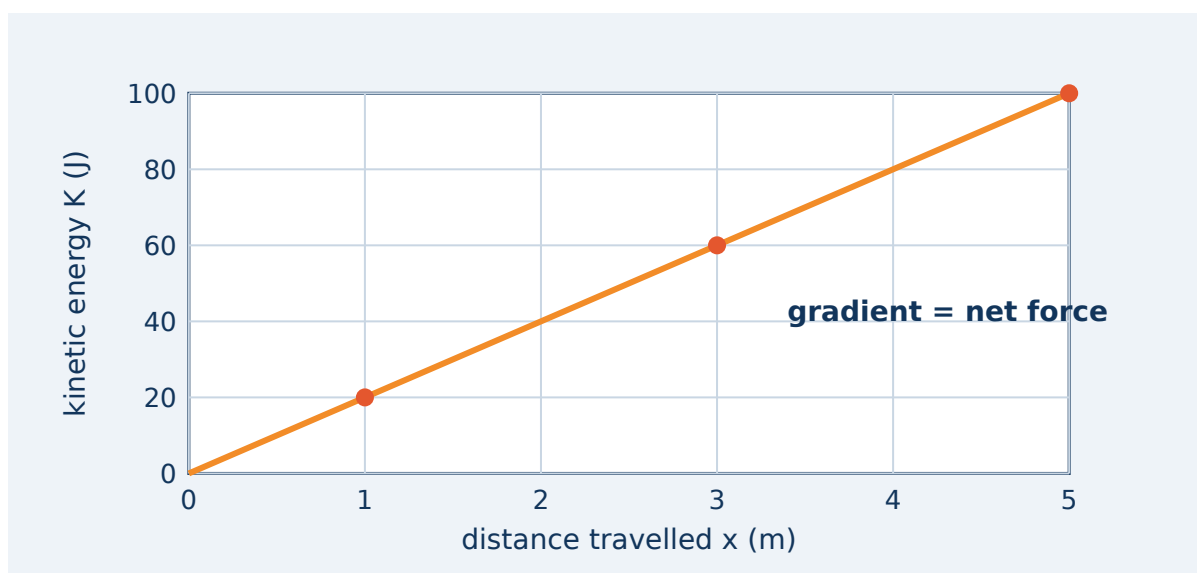
Worked example 3

A student pulls a trolley with a force that rises from 0 to 40 N over 0.50 m and then stays at 40 N for a further 0.50 m. **Find the total work.**

Triangle: $\frac{1}{2} \times 0.50 \times 40 = 10 \text{ J}$. Rectangle: $0.50 \times 40 = 20 \text{ J}$. Total = **30 J**.

What does the graph of kinetic energy look like?

If the net force is constant and acts along the motion, the kinetic energy grows in proportion to the distance, because $\Delta K = F_{\text{net}} d$. The gradient of a K against x graph is therefore the net force.



With a constant net force, kinetic energy grows in proportion to distance.

Common mistake

Students often use the weight, or the applied force alone, as the net force. The theorem uses the work of all forces, including friction and drag. A box pushed at constant speed has zero net work even though the pusher does positive work.

Lina's tip

Draw a free-body diagram and mark the displacement. Then ask for each force whether it points along, across or against the motion. This gives the sign of every term before you touch the calculator.

Key points to remember

- Work: $W = F d \cos\theta$ (J); area under a force-position graph if F varies.
- Kinetic energy: $K = \frac{1}{2} m v^2$ (J), proportional to speed squared.
- Work-energy theorem: $W_{\text{net}} = \Delta K = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2$.
- A force perpendicular to the displacement does zero work; friction does negative work.

Work, kinetic energy and the net-work theorem: frequently asked questions

What is the formula for work?

For a constant force, $W = F d \cos\theta$, where θ is the angle between the force and the displacement. The unit is the joule. For a changing force, find the area under the force-position graph.

When is the work done by a force zero?

Work is zero when there is no displacement, or when the force is perpendicular to the displacement. The normal force on a sliding block and the tension on a ball in a horizontal circle both do zero work.

What does the work-energy theorem say?

The net work on an object equals its change in kinetic energy, $W_{\text{net}} = \Delta K = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2$. It links forces and distances directly to speeds without needing the time.

Why is kinetic energy proportional to speed squared?

For a constant net force, $v^2 = v_0^2 + 2 a d$, so the work $F d$ needed to change the speed depends on v^2 . Doubling the speed therefore needs four times the net work.

Can work be negative?

Yes. When a force has a component opposite to the displacement, as friction does on a sliding object, the work is negative. It removes kinetic energy from the object.