

Work, kinetic energy and the net-work theorem

AP Physics 1 (algebra-based) · Worksheet answers

Complete methods are given for each problem. Notice how the sign of every work term is decided from the angle first, then the numbers are inserted. Final results are in bold with sensible significant figures.

Level: AP Physics 1 (algebra-based) · **Curriculum:** AP P1 Unit 3

Problem 1 Sign of the work

1. Positive: gravity points down and the apple moves down.
2. Negative: friction points against the motion.
3. Zero: the normal force is perpendicular to the horizontal displacement.
4. Zero: the tension is perpendicular to the velocity at every point.

Positive, negative, zero, zero.

Problem 2 Market cart

1. $W = F d = 120 \times 18 = \mathbf{2.2 \times 10^3 \text{ J}}$ (2160 J).
2. $d = W \div F = 4500 \div 150 = \mathbf{30 \text{ m}}$.

Problem 3 Tennis serve

1. $K = \frac{1}{2} m v^2 = \frac{1}{2} \times 0.057 \times 45^2 = \mathbf{58 \text{ J}}$.
2. $K \propto v^2$, so doubling the speed gives $4 \times 57.7 = \mathbf{2.3 \times 10^2 \text{ J}}$.

Problem 4 Units of energy

1. $1 \text{ J} = 1 \text{ N m} = 1 (\text{kg m/s}^2) \times \text{m} = 1 \text{ kg m}^2/\text{s}^2$.
2. $2.4 \text{ kJ} = \mathbf{2400 \text{ J}}$.
3. $72 \text{ km/h} = 72 \div 3.6 = 20 \text{ m/s}$, so $K = \frac{1}{2} \times 1000 \times 20^2 = \mathbf{2.0 \times 10^5 \text{ J}}$.

Problem 5 Angle matters

$W = F d \cos\theta$ with $F d = 200 \text{ J}$.

1. $200 \times 1 = \mathbf{200 \text{ J}}$.
2. $200 \times 0.5 = \mathbf{100 \text{ J}}$.
3. $200 \times 0 = \mathbf{0 \text{ J}}$.
4. $200 \times (-1) = \mathbf{-200 \text{ J}}$.

Problem 6 From work to speed

1. By the theorem, $\Delta K = W_{\text{net}} = \mathbf{150 \text{ J}}$.
2. $\frac{1}{2} m v^2 = 150$, so $v = \sqrt{2 \times 150 \div 3.0} = \mathbf{10 \text{ m/s}}$.

Problem 7 Force along a rail

1. Area of the triangle = $\frac{1}{2} \times 4.0 \times 30 = \mathbf{60 \text{ J}}$.
2. Add the rectangle: $6.0 \times 30 = 180 \text{ J}$, so the total is $60 + 180 = \mathbf{240 \text{ J}}$.
3. $\frac{1}{2} m v^2 = 240$, so $v = \sqrt{2 \times 240 \div 5.0} = \mathbf{9.8 \text{ m/s}}$.

Problem 8 Loaded trolley

1. $W = 30 \times 12 \times \cos 35^\circ = \mathbf{2.9 \times 10^2 \text{ J}}$ (295 J).
2. $W = -14 \times 12 = \mathbf{-168 \text{ J}}$.
3. $W_{\text{net}} = 295 - 168 = 127 \text{ J}$. Then $v = \sqrt{2 \times 127 \div 15} = \mathbf{4.1 \text{ m/s}}$. Gravity and the normal force do zero work.

Problem 9 Frictionless ramp

1. The height lost is $2.5 \times \sin 30^\circ = 1.25 \text{ m}$, so $W = m g h = 0.30 \times 9.81 \times 1.25 = \mathbf{3.7 \text{ J}}$.
2. **Zero**, since the normal force is perpendicular to the motion.
3. $\frac{1}{2} m v^2 = 3.68$, so $v = \sqrt{2 \times 3.68 \div 0.30} = \mathbf{5.0 \text{ m/s}}$.

Problem 10 Skateboard rolling to rest

1. $K = \frac{1}{2} \times 60 \times 5.0^2 = \mathbf{750 \text{ J}}$.
2. $F d = \Delta K$, so $F = 750 \div 12 = \mathbf{62.5 \text{ N}}$ (about 63 N).
3. $\mu = F \div (m g) = 62.5 \div (60 \times 9.81) = \mathbf{0.11}$.

Problem 11 Curling stone

1. $K = \frac{1}{2} \times 19 \times 2.4^2 = \mathbf{55 \text{ J}}$ (54.7 J).
2. $f = K \div d = 54.7 \div 33 = \mathbf{1.7 \text{ N}}$.
3. $\mu = 1.66 \div (19 \times 9.81) = \mathbf{0.0089}$, very small because ice is slippery.

Problem 12 Kinetic energy against distance

1. Gradient = $100 \text{ J} \div 5.0 \text{ m} = \mathbf{20 \text{ N}}$.
2. $\frac{1}{2} m v^2 = 100 \text{ J}$, so $m = 2 \times 100 \div 6.0^2 = \mathbf{5.6 \text{ kg}}$.
3. $a = F \div m = 20 \div 5.56 = \mathbf{3.6 \text{ m/s}^2}$.

Problem 13 Doubling the speed

1. $F d = \frac{1}{2} m v^2$, so $d = \frac{1}{2} \times 1200 \times 15^2 \div 6000 = \mathbf{22.5 \text{ m}}$.

- $d = \frac{1}{2} \times 1200 \times 30^2 \div 6000 = \mathbf{90 \text{ m}}$.
- K depends on v^2 , so doubling the speed quadruples the kinetic energy and the distance ($4 \times 22.5 = 90 \text{ m}$), not doubles it.

Problem 14 Stretching a spring

- A straight line through the origin; the area under it is the work done by the stretching force.
- $W = \frac{1}{2} k x^2 = \frac{1}{2} \times 150 \times 0.20^2 = \mathbf{3.0 \text{ J}}$.
- $W = \frac{1}{2} \times 150 \times (0.40^2 - 0.20^2) = \frac{1}{2} \times 150 \times 0.12 = \mathbf{9.0 \text{ J}}$. The second half costs three times as much, because the force is larger there.

Problem 15 Ranking kinetic energies

- $K = \frac{1}{2} \times 0.15 \times 30^2 = 67.5 \text{ J}$.
- $K = \frac{1}{2} \times 65 \times 6.0^2 = 1170 \text{ J}$.
- $K = \frac{1}{2} \times 0.0040 \times 400^2 = 320 \text{ J}$.

Runner ($1.2 \times 10^3 \text{ J}$) > bullet ($3.2 \times 10^2 \text{ J}$) > ball (68 J).

Problem 16 Pushing a crate up a ramp

- $W = -m g \sin 20^\circ \times d = -25 \times 9.81 \times 0.342 \times 4.0 = \mathbf{-3.4 \times 10^2 \text{ J}}$ (−336 J).
- $f = \mu m g \cos 20^\circ = 0.25 \times 245 \times 0.940 = 57.6 \text{ N}$, so $W = \mathbf{-2.3 \times 10^2 \text{ J}}$ (−230 J).
- Equilibrium along the ramp: $F = 83.9 + 57.6 = \mathbf{1.4 \times 10^2 \text{ N}}$.
- $W = 141.5 \times 4.0 = \mathbf{5.7 \times 10^2 \text{ J}}$. Sum: $566 - 336 - 230 = 0$, because the speed does not change ($\Delta K = 0$).

Problem 17 Force that changes sign

- Triangle: $\frac{1}{2} \times 4.0 \times 12 = \mathbf{24 \text{ J}}$.
- Triangle below the axis: $\frac{1}{2} \times 2.0 \times (-6.0) = \mathbf{-6.0 \text{ J}}$.
- $K_i = \frac{1}{2} \times 2.0 \times 3.0^2 = 9.0 \text{ J}$; $K_f = 9.0 + 24 - 6.0 = 27 \text{ J}$; $v = \sqrt{2 \times 27 \div 2.0} = \mathbf{5.2 \text{ m/s}}$.
- At $x = 4.0 \text{ m}$, where the force changes sign and the net work is largest: $K = 9 + 24 = 33 \text{ J}$, so $v = \sqrt{33} = \mathbf{5.7 \text{ m/s}}$.

Problem 18 Constant velocity paradox

- $W = 40 \times 6.0 = \mathbf{240 \text{ J}}$.
- Friction equals 40 N and opposes the motion, so $W = \mathbf{-240 \text{ J}}$.
- The claim is wrong. Bilal does 240 J of positive work, but friction does −240 J, so the net work is zero. The theorem then gives $\Delta K = 0$, consistent with constant speed. The 240 J does not vanish: it is transferred to thermal energy of the box and the floor.

Problem 19 Design: work and kinetic energy

1. Measure the distance d travelled from rest with a metre rule and the speed v at the end with a light gate or video; measure the masses on a balance.
2. Tilt the track slightly until the cart rolls at constant speed when pushed, so the weight component along the track cancels friction.
3. Plot $\Delta K = \frac{1}{2} (m + M) v^2$ against $W = M g d$ for the system. The gradient should be **1.0**.
4. Residual friction or pulley friction removes some work, so ΔK is less than $M g d$ and the gradient comes out below 1.

Problem 20 Falling through the air

1. $W_g = 75 \times 9.81 \times 120 = \mathbf{8.8 \times 10^4 \text{ J}}$ (88 290 J); $\Delta K = \frac{1}{2} \times 75 \times 30^2 = \mathbf{3.4 \times 10^4 \text{ J}}$ (33 750 J).
2. $W_{\text{air}} = \Delta K - W_g = 33\,750 - 88\,290 = \mathbf{-5.5 \times 10^4 \text{ J}}$; average drag = $54\,540 \div 120 = \mathbf{4.5 \times 10^2 \text{ N}}$.
3. $\Delta K = 0$, so $W_{\text{air}} = -W_g = -75 \times 9.81 \times 300 = \mathbf{-2.2 \times 10^5 \text{ J}}$.