

Work as an integral and potential energy curves

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet

Name: _____ Date: _____ Class: _____

These problems build the habit of writing work as an integral, reading energy from graphs, and moving between force and potential energy by differentiation. They start with constant forces and springs, then move to variable forces, equilibrium, and potential curves. Level: AP Physics C: Mechanics unit 3 and first-year university.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 3

Problem 1 Constant force work (easy)

A dock worker pulls a crate with a 40 N rope force through 5.0 m along level ground.

1. Calculate the work if the rope is parallel to the ground.
2. Calculate the work if the rope makes 60° with the ground.

Problem 2 Units of work (easy)

1. Show that the area under an F-x graph has the unit of joule.
2. Express the joule in base SI units.

Problem 3 Stretching a spring (easy)

Emma in Leeds stretches a spring with $k = 200 \text{ N/m}$ by 0.30 m from its natural length.

1. Calculate the work she does.

Problem 4 Work from rest (easy)

A net work of 120 J is done on a 3.0 kg trolley that starts at rest.

1. Calculate its final speed.

Problem 5 Force from a spring potential (easy)

The potential energy of a spring is $U = \frac{1}{2}kx^2$ with $k = 50 \text{ N/m}$.

1. Use $F = -dU/dx$ to find the force.
2. Calculate the force at $x = 0.20 \text{ m}$.

Problem 6 Conservative force (easy)

1. Define a conservative force.
2. State whether kinetic friction is conservative and give a reason.

Problem 7 Linear force (medium)

A force $F = 4.0x$ newtons acts from $x = 0$ to $x = 2.5$ m.

1. Calculate the work by integration.
2. Check using the area of a triangle.

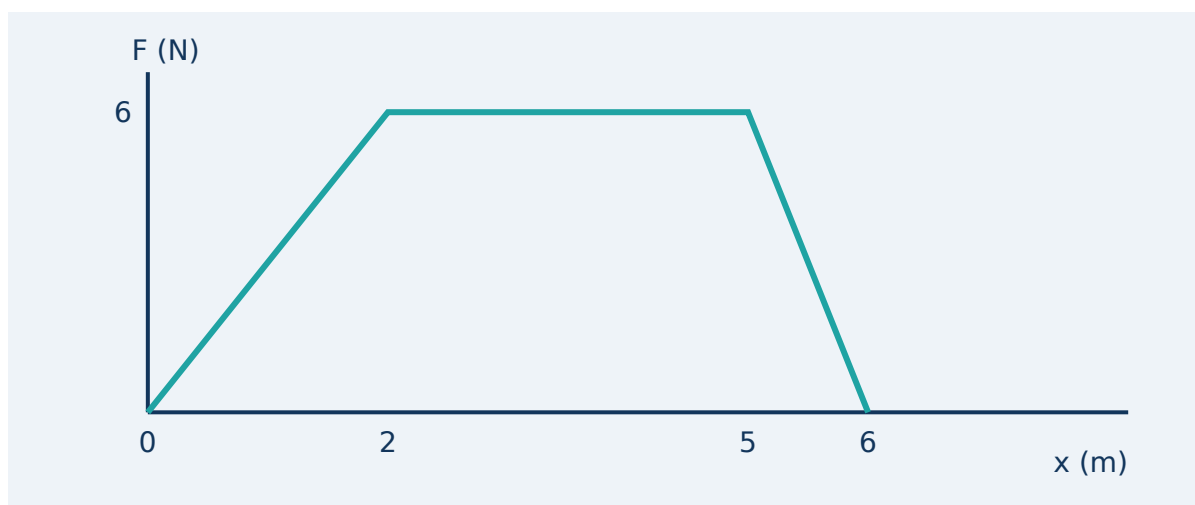
Problem 8 Force with an offset (medium)

Kofi in Accra pulls a 1.5 kg sled from rest with $F = (5.0 + 2.0x)$ N over 0 to 4.0 m. Ignore friction.

1. Calculate the work.
2. Calculate the final speed.

Problem 9 Piecewise force graph (medium)

The graph shows the force on a trolley as it moves along a track.



Force on a trolley against position.

1. Calculate the work from 0 to 2 m.
2. Calculate the work from 2 m to 5 m.
3. Calculate the total work from 0 to 6 m.

Problem 10 Force from a cubic potential (medium)

A particle has $U(x) = 2.0x^3 - 6.0x$ in joules, x in metres.

1. Find $F(x)$.
2. Calculate F at $x = 2.0$ m.
3. Find the equilibrium positions and classify them.

Problem 11 Motorbike power (medium)

A motorbike travels at a steady 20 m/s against a total resistive force of 600 N.

1. Calculate the power of the engine.
2. Calculate the work done in 30 s.

Problem 12 Gravity from potential (medium)

The gravitational potential energy of a mass m at distance r from Earth's centre is $U = -GMm/r$, with $GM = 3.99 \times 10^{14} \text{ m}^3/\text{s}^2$.

1. Find $F(r)$.
2. Calculate the force on 1.0 kg at $r = 6.37 \times 10^6 \text{ m}$.

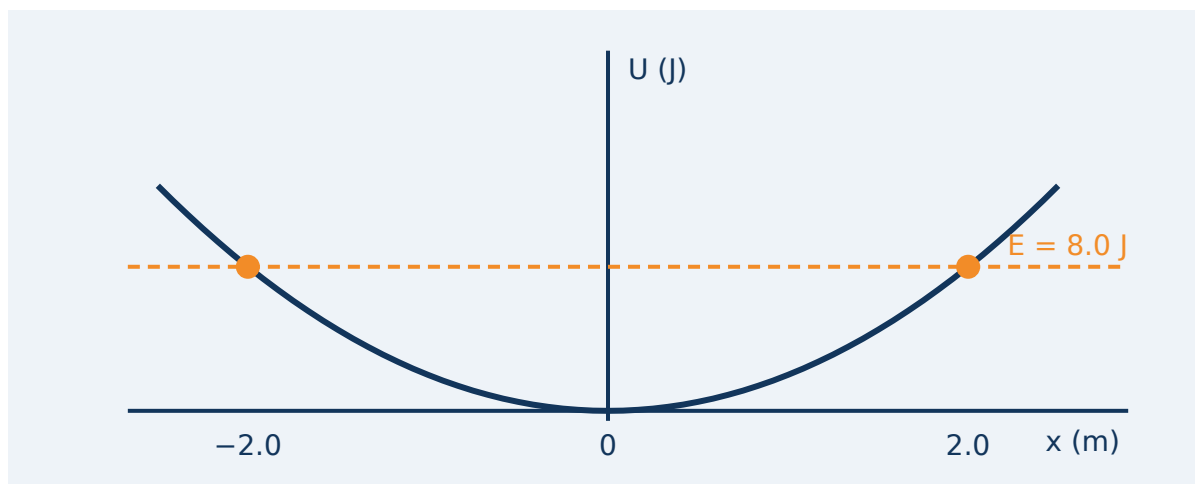
Problem 13 Average force mistake (medium)

Bilal in Lahore says: "The work of $F = 3.0x^2$ from 0 to 2.0 m is the average force times the distance, so $(0 + 12)/2 \times 2.0 = 12 \text{ J}$."

1. Identify the error.
2. Calculate the correct work.

Problem 14 Reading an energy diagram (medium)

A 0.50 kg particle moves in the potential $U = 2.0x^2 \text{ J}$ with total energy $E = 8.0 \text{ J}$.



Parabolic well with the energy line.

1. Find the turning points.
2. Calculate the maximum speed.
3. State where the maximum speed occurs.

Problem 15 Lifting out of the Earth's field (hard)

Mei in Singapore asks how much work is needed to lift a 1.0 kg mass from Earth's surface ($R = 6.37 \times 10^6 \text{ m}$) to a height equal to R . Use $GM = 3.99 \times 10^{14} \text{ m}^3/\text{s}^2$.

1. Write W as an integral and evaluate it.

2. Compare with mgh using $g = 9.8 \text{ m/s}^2$ and $h = R$.

Problem 16 Escape speed (hard)

Using the same data, derive and calculate the speed needed for a projectile to escape Earth from the surface, ignoring air.

1. Find the work to take 1 kg to infinity.
2. Equate with kinetic energy to find v .

Problem 17 Molecular potential (hard)

The interaction of two atoms is modelled by $U(r) = U_0[(a/r)^{12} - 2(a/r)^6]$, with $U_0 = 1.0 \times 10^{-21} \text{ J}$ and $a = 3.0 \times 10^{-10} \text{ m}$.

1. Show that $r = a$ is an equilibrium separation and give U there.
2. Calculate the effective spring constant for small oscillations, $k = d^2U/dr^2$ at $r = a$.

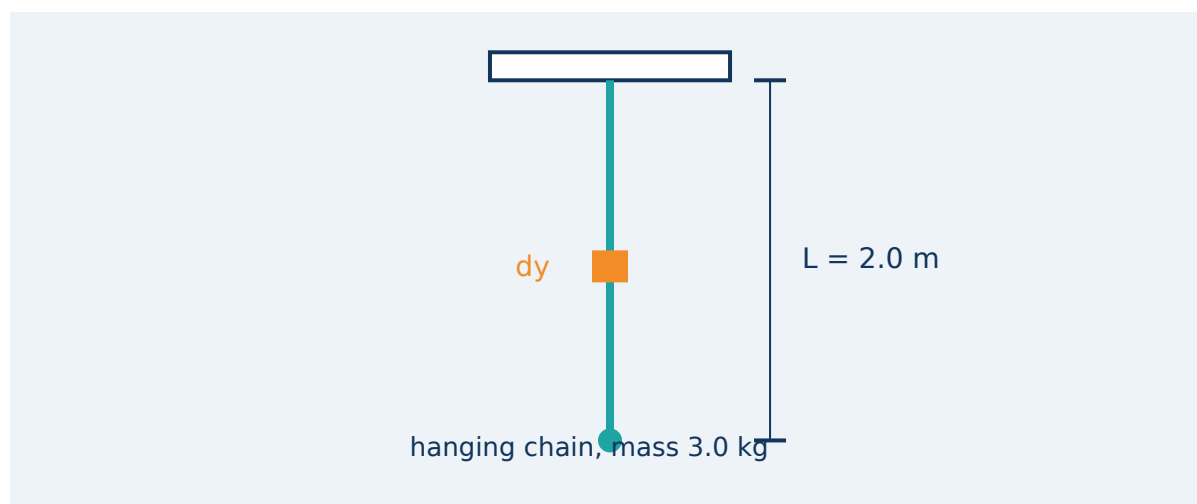
Problem 18 Sliding in a double well (hard)

A 1.0 kg bead moves on a wire with potential $U = x^4 - 4.0x^2$ joules (x in metres). It is released from rest at $x = 0.50 \text{ m}$.

1. Calculate F at the release point and the initial direction.
2. Calculate its speed when it reaches $x = +\sqrt{2}$.

Problem 19 Lifting a chain (hard)

A uniform chain of mass 3.0 kg and length 2.0 m hangs vertically from a table edge. Wanjiru lifts it slowly onto the table.



Chain hanging vertically.

1. Write the mass per unit length.
2. Write the work to lift the element at depth y by y .
3. Integrate to find the total work.

Problem 20 Force changing in time (hard)

A 2.0 kg particle starts from rest and feels $F = 6.0t$ newtons along x.

1. Find $v(t)$ by integrating F/m .
2. Find the power $P(t) = Fv$.
3. Integrate the power from 0 to 2.0 s and compare with the kinetic energy gained.