

Work as an integral and potential energy curves

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet answers

Each answer shows the integral or derivative, the substitution and the result with its unit. For sign questions the positive direction is stated. Use $g = 9.81 \text{ m/s}^2$ unless the problem gives another value, and keep two or three significant figures in final answers.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 3

Problem 1 Constant force work

1. $W = Fd \cos 0^\circ = 40 \times 5.0 = \mathbf{2.0 \times 10^2 \text{ J}}$.
2. $W = Fd \cos 60^\circ = 40 \times 5.0 \times 0.50 = \mathbf{1.0 \times 10^2 \text{ J}}$.

Problem 2 Units of work

1. $\text{N} \times \text{m} = \text{N} \cdot \text{m}$, which is the definition of the **joule**.
2. $\text{N} = \text{kg} \cdot \text{m/s}^2$, so $\text{J} = \mathbf{\text{kg} \cdot \text{m}^2/\text{s}^2}$.

Problem 3 Stretching a spring

$$W = \frac{1}{2}kx^2 = \frac{1}{2} \times 200 \times (0.30)^2 = \mathbf{9.0 \text{ J}}$$

Problem 4 Work from rest

$$W = \frac{1}{2}mv^2, \text{ so } v = \sqrt{(2W/m)} = \sqrt{(240/3.0)} = \sqrt{80} = \mathbf{8.9 \text{ m/s}}$$

Problem 5 Force from a spring potential

1. $dU/dx = kx$, so $F = -kx$.
2. $F = -50 \times 0.20 = \mathbf{-10 \text{ N}}$ (towards the equilibrium position).

Problem 6 Conservative force

1. A force whose work between two points is independent of the path, so a potential energy can be defined.
2. **No.** Friction's work depends on the path length, and the energy is converted to thermal energy.

Problem 7 Linear force

1. $W = \int_0^{2.5} 4.0x \, dx = 2.0x^2 = 2.0 \times 6.25 = \mathbf{12.5 \, J}$.
2. F at 2.5 m is 10 N, so area = $\frac{1}{2} \times 2.5 \times 10 = 12.5 \, \text{J}$, **in agreement**.

Problem 8 Force with an offset

1. $W = \int_0^4 (5.0 + 2.0x) \, dx = 5.0x + x^2 = 20 + 16 = \mathbf{36 \, J}$.
2. $v = \sqrt{(2W/m)} = \sqrt{(72/1.5)} = \sqrt{48} = \mathbf{6.9 \, m/s}$.

Problem 9 Piecewise force graph

1. Triangle: $\frac{1}{2} \times 2 \times 6 = \mathbf{6 \, J}$.
2. Rectangle: $3 \times 6 = \mathbf{18 \, J}$.
3. The last triangle is $\frac{1}{2} \times 1 \times 6 = 3 \, \text{J}$, so the total is $6 + 18 + 3 = \mathbf{27 \, J}$.

Problem 10 Force from a cubic potential

1. $F = -dU/dx = \mathbf{-6.0x^2 + 6.0}$ newtons.
2. $F(2.0) = -24 + 6.0 = \mathbf{-18 \, N}$.
3. $F = 0$ at $x = \pm 1.0 \, \text{m}$. $d^2U/dx^2 = 12x$, which is +12 at $x = 1.0$ (stable) and -12 at $x = -1.0$ (unstable).

Problem 11 Motorbike power

1. $P = Fv = 600 \times 20 = \mathbf{1.2 \times 10^4 \, W}$.
2. $W = Pt = 12\,000 \times 30 = \mathbf{3.6 \times 10^5 \, J}$.

Problem 12 Gravity from potential

1. $F = -dU/dr = \mathbf{-GMm/r^2}$ (attractive).
2. $F = 3.99 \times 10^{14} / (6.37 \times 10^6)^2 = 3.99 \times 10^{14} / 4.058 \times 10^{13} = \mathbf{9.8 \, N}$ (magnitude).

Problem 13 Average force mistake

1. The average of the end values is only correct when F is *linear* in x . $F = 3.0x^2$ is curved.
2. $W = \int_0^2 3.0x^2 \, dx = x^3 = \mathbf{8.0 \, J}$.

Problem 14 Reading an energy diagram

1. $U = E$ gives $2.0x^2 = 8.0$, so $x = \mathbf{\pm 2.0 \, m}$.
2. Maximum $K = E - U_{\min} = 8.0 \, \text{J}$, so $v = \sqrt{(2 \times 8.0/0.50)} = \sqrt{32} = \mathbf{5.7 \, m/s}$.
3. At $x = 0$, the bottom of the well, where U is smallest.

Problem 15 Lifting out of the Earth's field

1. $W = \int GMm/r^2 dr$ from R to $2R = GMm(1/R - 1/2R) = GMm/(2R) = 3.99 \times 10^{14}/(2 \times 6.37 \times 10^6) = \mathbf{3.1 \times 10^7 J}$.
2. $mgR = 1.0 \times 9.8 \times 6.37 \times 10^6 = 6.2 \times 10^7 J$, about **twice as large**, because g falls with height.

Problem 16 Escape speed

1. $W = \int$ from R to ∞ of $GMm/r^2 dr = GMm/R = 3.99 \times 10^{14}/6.37 \times 10^6 = 6.26 \times 10^7 J$ per kg.
2. $\frac{1}{2}v^2 = GM/R$, so $v = \sqrt{(2GM/R)} = \sqrt{(1.25 \times 10^8)} = \mathbf{1.1 \times 10^4 m/s}$.

Problem 17 Molecular potential

1. $dU/dr = U_0[-12a^{12}r^{-13} + 12a^6r^{-7}]$, which is 0 at $r = a$. $U(a) = U_0(1 - 2) = \mathbf{-1.0 \times 10^{-21} J}$.
2. $d^2U/dr^2 = U_0[156a^{12}r^{-14} - 84a^6r^{-8}] = 72U_0/a^2$ at $r = a$, so $k = 72 \times 1.0 \times 10^{-21}/(9.0 \times 10^{-20}) = \mathbf{0.80 N/m}$.

Problem 18 Sliding in a double well

1. $F = -dU/dx = -4x^3 + 8x = -0.5 + 4.0 = \mathbf{+3.5 N}$, towards $+x$.
2. $E = U(0.50) = 0.0625 - 1.0 = -0.9375 J$. $U(\sqrt{2}) = 4 - 8 = -4.0 J$, so $K = 3.0625 J$ and $v = \sqrt{(2 \times 3.0625/1.0)} = \mathbf{2.5 m/s}$.

Problem 19 Lifting a chain

1. $\lambda = 3.0/2.0 = 1.5 \text{ kg/m}$.
2. $dW = (\lambda dy) g y$.
3. $W = \int_0^L \lambda g y dy = \frac{1}{2}\lambda g L^2 = \frac{1}{2}mgL = 0.5 \times 3.0 \times 9.81 \times 2.0 = \mathbf{29 J}$.

Problem 20 Force changing in time

1. $v = \int (6.0t/2.0) dt = 1.5t^2 \text{ m/s}$.
2. $P = Fv = 6.0t \times 1.5t^2 = 9.0t^3 \text{ W}$.
3. $W = \int_0^2 9.0t^3 dt = (9.0/4)(16) = \mathbf{36 J}$. $K = \frac{1}{2} \times 2.0 \times (1.5 \times 4)^2 = 36 J$, so the work-energy theorem is confirmed.