

Work done, energy conservation and power

Physics grades 11-12 (A Level · IB · Class 11-12) · Worksheet answers

Each answer shows the formula, the substitution and the result with its unit. Compare your method first, because a correct method with a small arithmetic slip earns most of the credit in an exam. Round only at the end.

Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** IB A.3

Problem 1 Key relationships

1. $W = F s \cos \theta$.
2. $KE = \frac{1}{2} m v^2$ and $\Delta E_p = m g \Delta h$.
3. $P = W \div t$ and $P = F v$.
4. Efficiency = useful output $\div$ total input $\times 100 \%$.

$W = F s \cos \theta$, $KE = \frac{1}{2} m v^2$, $\Delta E_p = m g \Delta h$, $P = W/t = F v$.

Problem 2 Wheelbarrow of bricks

1. $W = F s = 140 \times 25 = 3500 \text{ J}$.
2. $3500 \text{ J} \div 1000 = 3.5 \text{ kJ}$.

$W = 3500 \text{ J} = 3.5 \text{ kJ}$.

Problem 3 Convert, then calculate

1. $72 \text{ km/h} = 72\,000 \text{ m} \div 3600 \text{ s} = 20 \text{ m/s}$.
2. $3.6 \text{ MJ} \div 3.6 \text{ MJ per kWh} = 1.0 \text{ kWh}$.
3. $E = P t = 0.450 \text{ kW} \times 2.0 \text{ h} = 0.90 \text{ kWh}$, and $0.90 \times 3.6 = 3.24 \text{ MJ}$.

20 m/s ; 1.0 kWh ; $0.90 \text{ kWh} = 3.2 \text{ MJ}$.

Problem 4 A footballer's kick

1. $KE = \frac{1}{2} m v^2 = \frac{1}{2} \times 0.43 \times 24^2 = 0.215 \times 576 = 123.8 \text{ J}$.
 2. KE is proportional to v^2 , so doubling the speed multiplies it by 4: $4 \times 123.84 = 495 \text{ J}$.
- Check: $\frac{1}{2} \times 0.43 \times 48^2 = 0.215 \times 2304 = 495.4 \text{ J}$.

$KE = 124 \text{ J}$; at 48 m/s , $KE = 495 \text{ J}$.

Problem 5 Rice on the shelf

1. $\Delta E_p = m g \Delta h = 15 \times 9.81 \times 1.4 = 206 \text{ J}$.
2. The lifting force transfers this energy to the sack, so the work done is 206 J .
3. Doubling Δh doubles the gain: $15 \times 9.81 \times 2.8 = 412 \text{ J}$.

206 J ; 206 J ; 412 J .

Problem 6 Lift motor

1. $P = W \div t = 48\,000 \div 12 = 4000 \text{ W} = 4.0 \text{ kW}$.

2. Efficiency = $4.0 \div 6.0 \times 100 \% = 66.7 \%$.

4.0 kW; efficiency = 67 %.

Problem 7 Dragging a sled

1. $F \cos \theta = 65 \times \cos 40^\circ = 65 \times 0.766 = 49.8 \text{ N}$.

2. $W = F s \cos \theta = 49.8 \times 18 = 896 \text{ J}$.

3. A smaller angle makes $\cos \theta$ larger, so a bigger part of the force acts along the motion. At 0° the full 65 N is used.

W = 896 J.

Problem 8 Positive, negative or zero

1. Negative: friction points opposite to the motion.

2. Zero: gravity is perpendicular to the displacement.

3. Zero: tension is always perpendicular to the velocity.

4. Positive: gravity points in the direction of motion.

5. $W = F s \cos 180^\circ = 35 \times 6.0 \times (-1) = -210 \text{ J}$.

-210 J.

Problem 9 Hockey stick strike

1. $W_{\text{net}} = \Delta KE = \frac{1}{2} \times 0.17 \times 30^2 - 0 = 76.5 \text{ J}$.

2. $F = W \div s = 76.5 \div 0.12 = 638 \text{ N}$.

W = 76.5 J; average F = $6.4 \times 10^2 \text{ N}$.

Problem 10 Force on a runway

Work equals the area under the graph.

1. Triangle: $\frac{1}{2} \times 2 \times 40 = 40 \text{ J}$.

2. Rectangle: $3 \times 40 = 120 \text{ J}$.

3. Final triangle: $\frac{1}{2} \times 2 \times 40 = 40 \text{ J}$, so the total is $40 + 120 + 40 = 200 \text{ J}$.

4. $\frac{1}{2} m v^2 = 200$, so $v = \sqrt{(2 \times 200 \div 4.0)} = \sqrt{100} = 10 \text{ m/s}$.

200 J; 10 m/s.

Problem 11 Swinging bob

1. $m g h = \frac{1}{2} m v^2$, so $v = \sqrt{(2 g h)} = \sqrt{(2 \times 9.81 \times 0.35)} = \sqrt{6.867} = 2.6 \text{ m/s}$.

2. $KE = m g h = 0.50 \times 9.81 \times 0.35 = 1.7 \text{ J}$.

3. The speed is unchanged because m cancels from both sides; only the kinetic energy doubles to 3.4 J.

v = 2.6 m/s; KE = 1.7 J.

Problem 12 Motorbike thrust

1. $P = F v$, so $F = P \div v = 18\,000 \div 25 = 720\text{ N}$.

2. $W = P t = 18\,000 \times 300 = 5.4 \times 10^6\text{ J}$.

F = 720 N; W = 5.4 MJ.

Problem 13 Solar water pump

1. $W = m g h = 540 \times 9.81 \times 6.0 = 31\,784\text{ J}$.

2. $P = 31\,784 \div 60 = 530\text{ W}$.

3. Efficiency = $530 \div 750 \times 100\% = 70.6\%$.

4. Friction in the pump and thermal energy in the motor and wires warm the surroundings.

$3.18 \times 10^4\text{ J}$; 530 W ; efficiency = 71% .

Problem 14 Braking distance

1. $KE = \frac{1}{2} \times 950 \times 15^2 = 475 \times 225 = 106\,875\text{ J}$.

2. The work done by the brakes equals the kinetic energy: $F d = KE$, so $d = 106\,875 \div 4750 = 22.5\text{ m}$.

3. KE at $30\text{ m/s} = \frac{1}{2} \times 950 \times 900 = 427\,500\text{ J}$, so $d = 427\,500 \div 4750 = 90\text{ m}$, four times as far.

$1.07 \times 10^5\text{ J}$; 22.5 m ; 90 m .

Problem 15 Find the errors

1. Error one: the force needed is the weight $m g$, not the mass, so g is missing. Error two: power is work divided by time, not multiplied.

2. $W = m g h = 20 \times 9.81 \times 3.0 = 588.6\text{ J}$, and $P = W \div t = 588.6 \div 4.0 = 147\text{ W}$.

W = 589 J; P = $1.5 \times 10^2\text{ W}$.

Problem 16 Sliding down a slope

1. $h = s \sin 30^\circ = 5.0 \times 0.50 = 2.5\text{ m}$.

2. $\Delta E_p = m g h = 8.0 \times 9.81 \times 2.5 = 196.2\text{ J}$.

3. $W_f = 12 \times 5.0 = 60\text{ J}$.

4. $KE = 196.2 - 60 = 136.2\text{ J}$, so $v = \sqrt{(2 \times 136.2 \div 8.0)} = \sqrt{34.05} = 5.8\text{ m/s}$.

196 J lost; 60 J against friction; $v = 5.8\text{ m/s}$.

Problem 17 A bouncing ball

1. $v = \sqrt{(2 g h)} = \sqrt{(2 \times 9.81 \times 1.5)} = \sqrt{29.43} = 5.4\text{ m/s}$.

2. $v = \sqrt{(2 \times 9.81 \times 1.1)} = \sqrt{21.58} = 4.6\text{ m/s}$.

3. Energy lost = $m g (1.5 - 1.1) = 0.060 \times 9.81 \times 0.40 = 0.235\text{ J}$.

4. Fraction lost = $0.40 \div 1.5 = 26.7\%$.

5.4 m/s; 4.6 m/s; 0.24 J lost; 27 %.

Problem 18 Lorry on a hill

1. $m g \sin \theta = 12\,000 \times 9.81 \times 0.050 = 5886 \text{ N}$.
 2. Constant speed means zero net force: driving force = $6000 + 5886 = 11\,886 \text{ N}$.
 3. $P = F v = 11\,886 \times 15 = 178\,290 \text{ W} = 178 \text{ kW}$.
 4. Efficiency = $178.3 \div 450 \times 100 \% = 39.6 \%$.
- $1.19 \times 10^4 \text{ N}$; 178 kW ; efficiency = 40% .**

Problem 19 Lift with a load

1. $\Delta E_p = 1500 \times 9.81 \times 30 = 441\,450 \text{ J}$.
 2. $\Delta KE = \frac{1}{2} \times 1500 \times 2.0^2 = 3000 \text{ J}$.
 3. $W = 441\,450 + 3000 = 444\,450 \text{ J}$.
 4. $P = 444\,450 \div 18 = 24\,692 \text{ W} = 24.7 \text{ kW}$.
 5. Input = $24.69 \div 0.85 = 29.0 \text{ kW}$.
- $W = 4.44 \times 10^5 \text{ J}$; useful $P = 24.7 \text{ kW}$; input = 29.0 kW .**

Problem 20 Wind turbine power

1. In one second a column of air of length v and volume $A v$ passes, with mass $\rho A v$. Its kinetic energy is $\frac{1}{2} (\rho A v) v^2 = \frac{1}{2} \rho A v^3$, so $P = \frac{1}{2} \rho A v^3$.
 2. $A = \pi \times 20^2 = 1257 \text{ m}^2$. $P = \frac{1}{2} \times 1.2 \times 1257 \times 8.0^3 = 0.6 \times 1257 \times 512 = 3.86 \times 10^5 \text{ W}$.
 3. Efficiency = $150 \div 386 \times 100 \% = 38.9 \%$.
 4. P is proportional to v^3 , so doubling the speed multiplies it by 8: $8 \times 386 = 3088 \text{ kW}$.
- 386 kW ; efficiency = 39% ; 3.09 MW .**