

Springs, wires and the stiffness of materials

Physics grades 11-12 (A Level · IB · Class 11-12) · Lesson

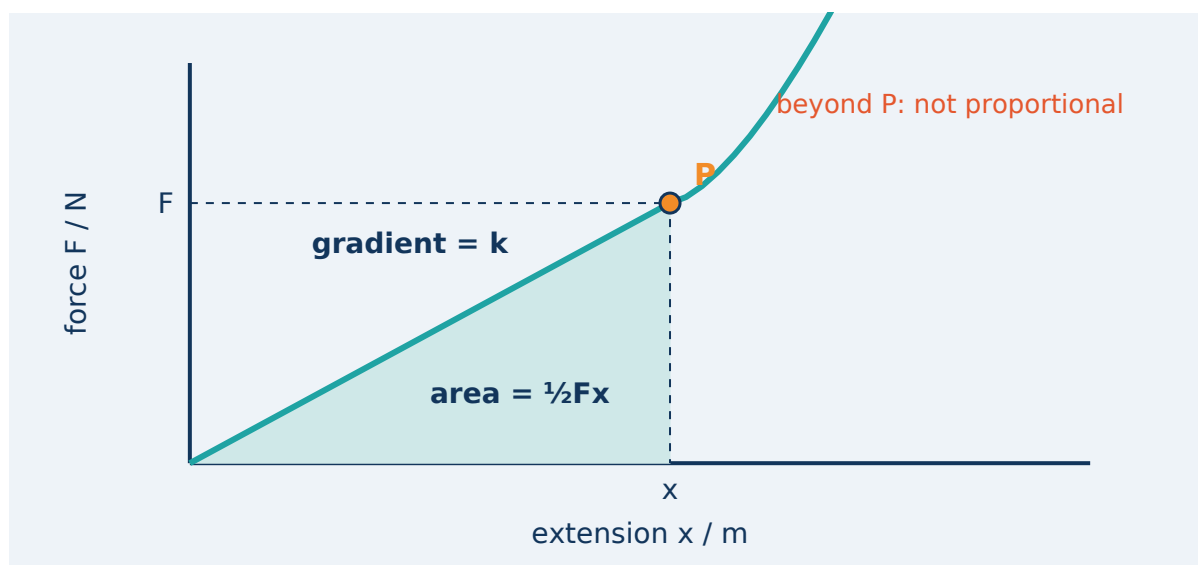
Quick answer: Hooke's law says extension is proportional to force ($F = kx$) up to the limit of proportionality. Stress $\sigma = F/A$ and strain $\epsilon = x/L$ describe the material, and the Young modulus $E = \sigma/\epsilon$ is the gradient of the straight part of the stress-strain graph. Stored energy is $\frac{1}{2}Fx$.

Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** A Level 3.4.2

What is Hooke's law?

Hooke's law Hooke's law states that the extension of a spring or wire is directly proportional to the force applied, provided the limit of proportionality is not exceeded: **$F = kx$** .

The constant k is the *force constant* (or stiffness) of the spring, measured in N/m. The graph of force against extension is a straight line through the origin with gradient k . Beyond the limit of proportionality the line curves and Hooke's law no longer applies. A stiffer spring has a larger k .



Force against extension. The straight part obeys Hooke's law and the shaded area is the energy stored.

How much energy is stored in a stretched spring or wire?

The force grows from zero to F as the extension grows from zero to x , so the average force is $F/2$. The work done, which is stored as elastic potential energy, equals the area under the force-extension graph.

Elastic potential energy

$$E = \frac{1}{2}Fx = \frac{1}{2}kx^2 = \frac{F^2}{2k}$$

E is the elastic potential energy (J), F the force (N), x the extension (m), k the force constant (N/m). Valid only inside the elastic limit and where Hooke's law applies.

Worked example 1

A spring has $k = 250 \text{ N/m}$. Find the energy stored at an extension of 0.060 m and the extra energy needed to stretch it to 0.100 m .

E at $0.060 \text{ m} = \frac{1}{2} \times 250 \times 0.060^2 = \mathbf{0.45 \text{ J}}$. E at $0.100 \text{ m} = \frac{1}{2} \times 250 \times 0.100^2 = 1.25 \text{ J}$.
Extra energy = $1.25 - 0.45 = \mathbf{0.8 \text{ J}}$.

What are stress, strain and the Young modulus?

A thick wire stretches less than a thin wire of the same material, and a long wire stretches more than a short one. Stress and strain remove these effects to describe the *material* itself.

Stress, strain and Young modulus

Stress $\sigma = F/A$, force per unit cross-sectional area ($\text{Pa} = \text{N/m}^2$). **Strain** $\epsilon = x/L$, extension per unit original length (no unit). The **Young modulus** $E = \sigma/\epsilon$, measured in pascals, is the gradient of the straight part of a stress-strain graph.

Material	Young modulus E / GPa	Typical behaviour
Steel	200	Stiff and ductile
Copper	120	Ductile
Aluminium	70	Ductile, lighter
Glass	70	Brittle
Nylon	3	Flexible polymer
Rubber	about 0.01	Stretches a great deal

Wire formulas

$E = (F/A) \div (x/L) = FL/Ax$, and the force constant of a wire is $k = EA/L$.

F is the load (N), A the cross-sectional area (m^2), L the original length (m), x the extension (m).

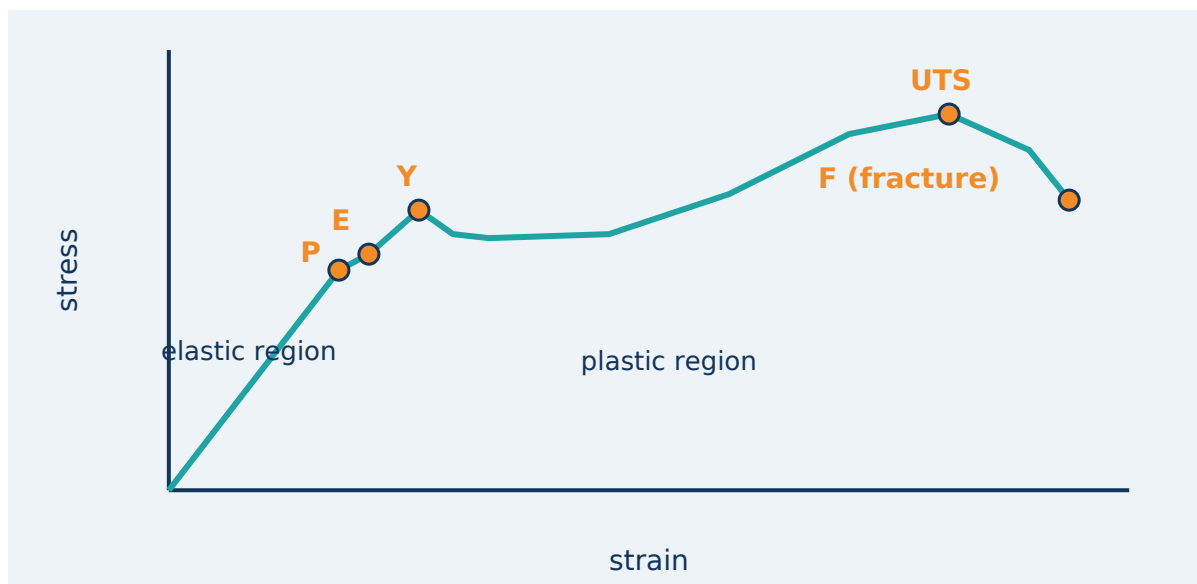
Worked example 2

A steel wire 2.50 m long and 0.80 mm in diameter carries a 150 N load ($E = 2.0 \times 10^{11} \text{ Pa}$). Calculate the stress, strain and extension.

$A = \pi(0.40 \times 10^{-3})^2 = 0.000 \text{ m}^2$. Stress = $F/A = 150 \div 0.000 = \mathbf{298415518.297 \text{ Pa}}$. Strain = $\sigma/E = 298415518.297 \div 2.0 \times 10^{11} = \mathbf{0.00149}$. Extension = strain $\times L = 0.00149 \times 2.50 = \mathbf{3.73 \text{ mm}}$.

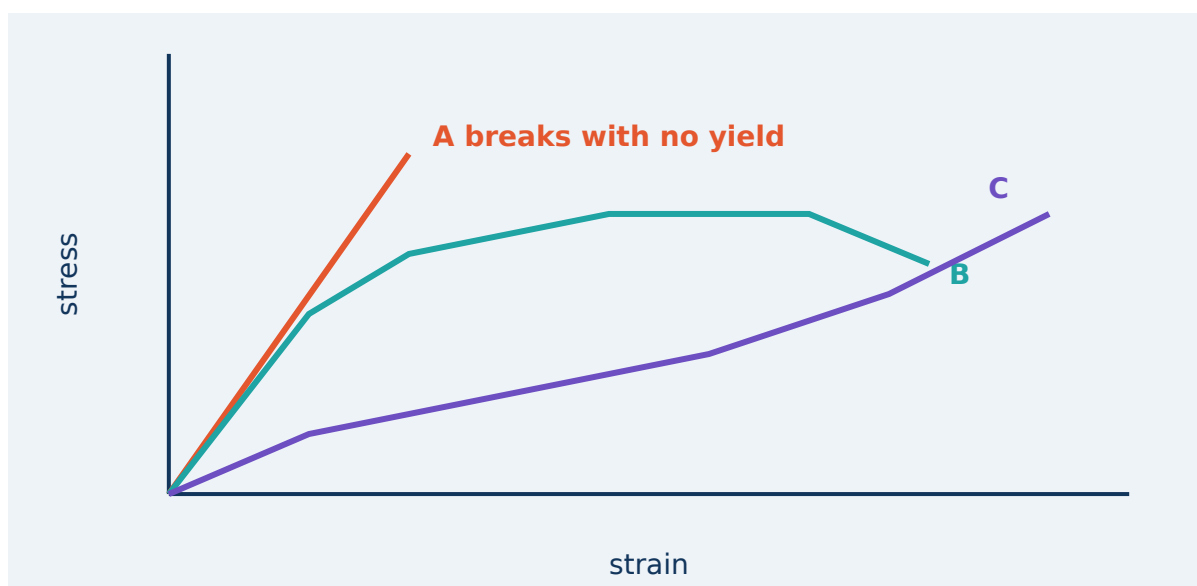
What does a stress-strain graph tell you?

On the straight part, up to the limit of proportionality (P), stress is proportional to strain. A little beyond is the elastic limit (E): up to here the material returns to its original length when the load is removed. After the yield point (Y) the metal stretches a lot with little extra stress and deformation becomes **plastic**, meaning permanent. The highest point is the ultimate tensile stress (UTS). The wire then narrows (necks) and breaks at the fracture point F.



Stress-strain graph for a ductile metal such as mild steel.

Different materials give very different curves. A **brittle** material such as glass or ceramic follows a steep straight line and breaks with almost no plastic region. A **ductile** material such as copper can be drawn into wire. A **polymer** such as nylon has a low gradient and a long, curved plastic region.



Three material types: brittle (A), ductile (B) and polymeric (C).

Common mistake

Students confuse stiffness with strength. The Young modulus (gradient) measures stiffness; the UTS (highest stress) measures strength. A material can be stiff and brittle,

like glass, or flexible and strong.

Lina's tip

Convert diameter to area early: $A = \pi(d/2)^2$. Half the diameter first, then square it. Doubling the diameter makes the area four times bigger.

Key points to remember

- Hooke's law: $F = kx$, k in N/m, valid up to the limit of proportionality.
- Elastic potential energy $E = \frac{1}{2}Fx = \frac{1}{2}kx^2$, the area under the F - x graph.
- Stress $\sigma = F/A$ (Pa); strain $\epsilon = x/L$ (no unit).
- Young modulus $E = \sigma/\epsilon = FL/Ax$; steel is about 200 GPa.
- Elastic deformation is reversible; plastic deformation is permanent.

Springs, wires and the stiffness of materials: frequently asked questions

What is the difference between the limit of proportionality and the elastic limit?

The limit of proportionality is where force stops being proportional to extension. The elastic limit is the largest stress after which the material returns to its original length on unloading. They are usually very close in metals.

What is the unit of the Young modulus?

The unit is the pascal (Pa), because strain has no unit and the modulus equals stress divided by strain. Values for metals are usually quoted in gigapascals ($1 \text{ GPa} = 10^9 \text{ Pa}$).

Why is the energy stored $\frac{1}{2}Fx$ and not Fx ?

The force rises steadily from zero to F as the extension grows, so the average force over the stretch is $F/2$. The work done is the average force multiplied by the extension.

What is the difference between ductile and brittle?

A ductile material shows a large plastic region and can be stretched into wire without breaking. A brittle material breaks soon after the elastic limit with little or no plastic deformation.

How can you find the Young modulus in the lab?

Hang increasing loads on a long thin wire and measure each extension. Plot force against extension, find the gradient and use $E = (\text{gradient}) \times L \div A$, with the area from the measured diameter.