

Springs, wires and the stiffness of materials

Physics grades 11-12 (A Level · IB · Class 11-12) · Worksheet answers

Each answer shows the formula, the substitution and the result with its unit in bold. Use $E = 200 \text{ GPa}$ for steel, 120 GPa for copper and 70 GPa for aluminium where a modulus is not given, and take $g = 9.81 \text{ N/kg}$.

Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** A Level 3.4.2

Problem 1 Hooke's law in words

1. The extension of a spring is **directly proportional to the applied force**, provided the limit of proportionality is not exceeded.
2. The unit is **N/m**.
3. A straight line through the origin with gradient k .

Problem 2 Finding k

1. $k = F/x = 6.0 \div 0.030 = \mathbf{200 \text{ N/m}}$.
2. $F = kx = 200 \times 0.045 = \mathbf{9 \text{ N}}$.
3. $x = F/k = 15 \div 200 = \mathbf{0.075 \text{ m}}$ (7.5 cm), assuming the limit is not exceeded.

Problem 3 Stress in a wire

1. $2.0 \text{ mm}^2 = 2.0 \times (10^{-3} \text{ m})^2 = \mathbf{2.0 \times 10^{-6} \text{ m}^2}$.
2. $\sigma = F/A = 500 \div 2.0 \times 10^{-6} = \mathbf{250000000.000 \text{ Pa}}$.
3. $2.5 \times 10^8 \text{ Pa} = \mathbf{250 \text{ MPa}}$.

Problem 4 Strain of a cable

1. $1.5 \text{ mm} = \mathbf{0.0015 \text{ m}}$.
2. $\epsilon = x/L = 0.0015 \div 3.00 = \mathbf{0.0005}$ (no unit).
3. $0.05 \% = \mathbf{0.050 \%}$.

Problem 5 Young modulus from stress and strain

1. $E = \text{stress} \div \text{strain}$.
2. $E = 1.0 \times 10^8 \div 5.0 \times 10^{-4} = \mathbf{200000000000.000 \text{ Pa}}$.
3. $2.0 \times 10^{11} \text{ Pa} = \mathbf{200 \text{ GPa}}$, which is steel.

Problem 6 Reading a force-extension graph

1. The graph stops being straight at **8.0 N** (extension 0.040 m).

2. $k = F/x = 8.0 \div 0.040 = \mathbf{200 \text{ N/m}}$.
3. $x = 6.0 \div 200 = 0.030 \text{ m}$. $E = \frac{1}{2}Fx = \frac{1}{2} \times 6.0 \times 0.030 = \mathbf{0.09 \text{ J}}$.

Problem 7 Elastic or plastic?

1. The wire stays **longer than the original length**: a permanent extension remains.
2. In elastic deformation the material returns to its original shape when the load is removed; in plastic deformation some deformation is permanent.
3. Ductile: copper (or mild steel). Brittle: glass (or ceramic, cast iron).

Problem 8 Copper wire stretch

1. $A = \pi(0.25 \times 10^{-3})^2 = \mathbf{0.000 \text{ m}^2}$.
2. $\sigma = 30 \div 0.000 = \mathbf{152788745.368 \text{ Pa}}$. $\epsilon = \sigma/E = 152788745.368 \div 1.2 \times 10^{11} = \mathbf{0.00127}$.
3. $x = \epsilon L = 0.00127 \times 1.80 = 0.00229 \text{ m} = \mathbf{2.29 \text{ mm}}$.

Problem 9 Two wires compared

1. $x = FL/(AE)$ and $A \propto d^2$, so $x \propto L/d^2$.
2. B: L doubled ($\times 2$), $d^2 \times 4$, so $x = 4.0 \times 2 \div 4 = \mathbf{2.0 \text{ mm}}$.
3. d halved: A is $\div 4$, so $x = 4.0 \times 4 = \mathbf{16 \text{ mm}}$.

Problem 10 Catapult

1. $F = kx = 120 \times 0.15 = \mathbf{18 \text{ N}}$.
2. $E = \frac{1}{2}kx^2 = \frac{1}{2} \times 120 \times 0.15^2 = \mathbf{1.35 \text{ J}}$.
3. $\frac{1}{2}mv^2 = E$, so $v = \sqrt{(2 \times 1.35 \div 0.20)} = \mathbf{3.67 \text{ m/s}}$.

Problem 11 Energy stored in a wire

1. $E = \frac{1}{2}Fx = \frac{1}{2} \times 200 \times 1.0 \times 10^{-3} = \mathbf{0.1 \text{ J}}$.
2. $V = AL = 1.0 \times 10^{-6} \times 2.0 = \mathbf{2.0 \times 10^{-6} \text{ m}^3}$.
3. $0.1 \div 2.0 \times 10^{-6} = \mathbf{50000.000 \text{ J/m}^3}$. Check: $\sigma = 200 \div 1.0 \times 10^{-6} = 2.0 \times 10^8 \text{ Pa}$, $\epsilon = 1.0 \times 10^{-3} \div 2.0 = 5.0 \times 10^{-4}$, $\frac{1}{2}\sigma\epsilon = \frac{1}{2} \times 2.0 \times 10^8 \times 5.0 \times 10^{-4} = 50000.000 \text{ J/m}^3 \checkmark$.

Problem 12 Reading a stress-strain graph

1. Stress at P = **250 MPa**; UTS = **420 MPa**.
2. $E = 250 \times 10^6 \div 1.25 \times 10^{-3} = \mathbf{200000000000.000 \text{ Pa}}$ (200 GPa).
3. Breaking stress = **380 MPa**. The wire narrows (necks) before it breaks, so the actual area is smaller; the calculated stress uses the original area, so it falls even though the real local stress is higher.

Problem 13 Rubber band behaviour

1. **No**: the force-extension graph is curved, not a straight line through the origin.
2. The area under the loading curve is the work done on the band; the area under the unloading curve is the energy returned. The enclosed area is the **energy dissipated as thermal energy** inside the rubber.
3. The dissipated energy raises the internal energy of the rubber, so its temperature increases.

Problem 14 Same stress, different strain

1. steel: $\epsilon = \sigma/E = 1.0 \times 10^8 \div 200 \times 10^9 = 0.0005$
aluminium: $\epsilon = \sigma/E = 1.0 \times 10^8 \div 70 \times 10^9 = 0.00143$
nylon: $\epsilon = \sigma/E = 1.0 \times 10^8 \div 3 \times 10^9 = 0.0333$.
2. **Steel** has the largest Young modulus, so it is stiffest.
3. **Nylon** extends most. Factor = 66.7 — **about 67 times** that of steel.

Problem 15 Crane cable

1. $4.0 \text{ cm}^2 = 4.0 \times 10^{-4} \text{ m}^2$.
2. $F = \sigma A = 4.0 \times 10^8 \times 4.0 \times 10^{-4} = \mathbf{160000.000 \text{ N}}$ (160 kN).
3. Working load = $160000.000 \div 4 = 40000.000 \text{ N}$, so $m = F/g = 40000.000 \div 9.81 = \mathbf{4077.472 \text{ kg}}$.

Problem 16 Steel and copper in parallel

1. Same x , L and A , so $F = EA x/L$: F is proportional to E .
2. Steel takes $200 \div 320$ of the load: **500 N**. Copper takes **300 N**.
3. $x = FL/(EA) = 500 \times 1.0 \div (2.0 \times 10^{11} \times 1.0 \times 10^{-6}) = \mathbf{2.5 \text{ mm}}$. Check with copper: $300 \div (1.2 \times 10^{11} \times 1.0 \times 10^{-6}) = 2.5 \text{ mm} \checkmark$.

Problem 17 Sudden loading

1. $k = EA/L = 2.0 \times 10^{11} \times 1.0 \times 10^{-6} \div 1.5 = \mathbf{133333.3333 \text{ N/m}}$.
2. $x = mg/k = 98.1 \div 133333.3333 = \mathbf{0.736 \text{ mm}}$.
3. Loss of gravitational energy = gain in elastic energy: $mgx_{\max} = \frac{1}{2}kx_{\max}^2$, so $x_{\max} = 2mg/k = 2 \times 0.736 = \mathbf{1.47 \text{ mm}}$. $\sigma_{\max} = kx_{\max}/A = 133333.3333 \times 0.00147 \div 1.0 \times 10^{-6} = \mathbf{196200000.000 \text{ Pa}}$ (196 MPa).

Problem 18 Polymer strip data

1. The line is straight up to **8 N (40 mm)**. $k = 8 \div 0.040 = \mathbf{200 \text{ N/m}}$.
2. $E = \frac{1}{2} \times 8 \times 0.040 = \mathbf{0.16 \text{ J}}$.
3. 8-10 N: $\frac{1}{2}(8 + 10) \times (0.055 - 0.040) = 0.135 \text{ J}$. 10-12 N: $\frac{1}{2}(10 + 12) \times (0.078 - 0.055) = 0.253 \text{ J}$. Total = $0.16 + 0.135 + 0.253 = \mathbf{0.548 \text{ J}}$.

Problem 19 Young modulus from a gradient

1. Gradient = $F/x = EA/L$, so $E = \text{gradient} \times L/A$.
2. $A = \pi(0.45 \times 10^{-3})^2 = 0.000 \text{ m}^2$. $E = 1.86 \times 10^4 \times 2.4 \div 0.000 = \mathbf{70169646020.960 \text{ Pa}}$ (70 GPa, aluminium).
3. Diameter: $\pm 1.1 \%$, doubled for area = 2.2% . Length: $\pm 0.083 \%$. Total $\approx \mathbf{2.3 \%}$ (about $\pm 1.6 \text{ GPa}$). The **diameter** is the largest source because it is squared.

Problem 20 Evaluate the claims

1. The Young modulus is a property of the **material**, not of the wire's size. A thicker wire has the same E , but carries a bigger force for the same stress.
2. The units were mixed: $1.2 \text{ mm} = 1.2 \times 10^{-3} \text{ m}$, so $\epsilon = 1.2 \times 10^{-3} \div 2.0 = \mathbf{0.0006}$ (6.0×10^{-4}), not 0.6.
3. The force constant $k = EA/L$, which grows with the cross-sectional area A .